Module
7

Applied Mathematics 30

VECTORS

Learning
Technologies
Branch

Alberta
LEARNING

Module 7

Applied Mathematics 30

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Applied Mathematics 30
Module 7: Vectors
Student Module Booklet
Learning Technologies Branch
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This document is intended for	
Students	✓
Teachers	✓
Administrators	
Home Instructors	
General Public	
Other	

You may find the following Internet sites useful:

- Alberta Learning, **http://www.learning.gov.ab.ca**
- Learning Technologies Branch, **http://www.learning.gov.ab.ca/ltb**
- Learning Resources Centre, **http://www.lrc.learning.gov.ab.ca**

The use of the Internet is optional. Exploring the electronic information superhighway can be educational and entertaining. However, be aware that these computer networks are not censored. Students may unintentionally or purposely find articles on the Internet that may be offensive or inappropriate. As well, the sources of information are not always cited and the content may not be accurate. Therefore, students may wish to confirm facts with a second source.

Welcome

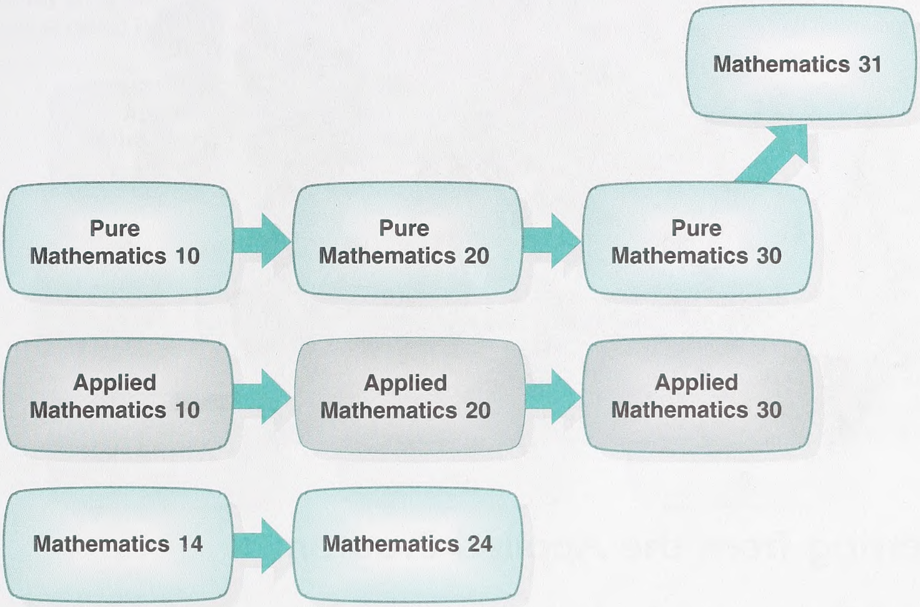
Applied Mathematics 30

Applied Mathematics 30 contains seven modules and a final test. Work through the modules in the order given, since several concepts build on each other as you progress through the course.

Contents

Introduction to Applied Mathematics 30

Applied Mathematics 30 is the third course in the Applied Mathematics 10–20–30 program of studies. Another program of studies is Pure Mathematics 10–20–30; students who complete Pure Mathematics 30 often choose to take Mathematics 31. A third program of studies is Mathematics 14–24.



Each mathematics program is designed for students with different mathematical strengths and interests.

- Pure Mathematics 10–20–30 is intended for students who are strong in algebra and mathematical theory.
- Applied Mathematics 10–20–30 is better suited to students who prefer to solve problems using numerical reasoning or geometry.
- Mathematics 14–24 is a general mathematics program for high school students who have experienced difficulties in previous mathematics courses.

Each sequence of courses is designed for students with different career plans. For example, Pure Mathematics 30 is a prerequisite for admission to many university programs. Many colleges and technical institutes, however, will admit students who have successfully completed Applied Mathematics 30.

You may find it helpful to read any of the documents under the heading "New Senior High School Mathematics Update/Post-Secondary Studies Update" at the following Internet site:

http://www.learning.gov.ab.ca/k_12/curriculum/bySubject/math

Before enrolling in Applied Mathematics 30, it is recommended that you talk with a school counsellor about your career plans.

Transferring from the Applied Program

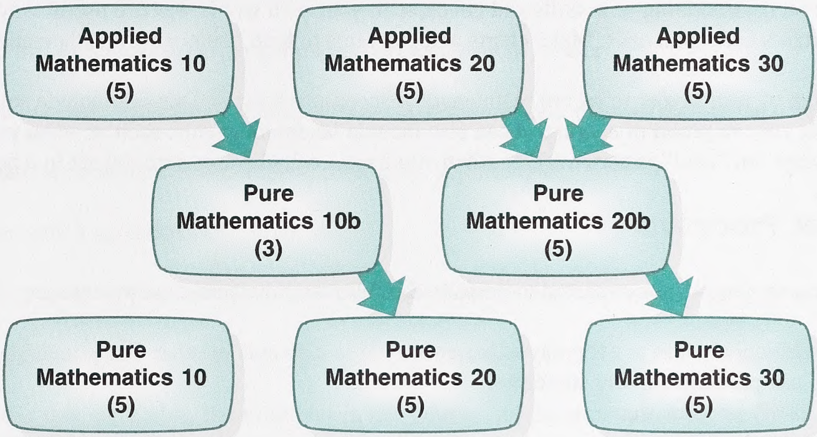
You should be aware that the applied and pure mathematics courses do have some topics in common; other topics are independent.

The following table shows some common and independent topics.

Applied Topics	Common Topics	Pure Topics
• linear programming • data tables and trends • design and layout • metric and imperial measure • data presentation • vectors and matrices • periodic, fractal, and recursive patterns • financial decision making • costing and design problems	• spreadsheets • line segments and linear graphs • scaling • triangles • financial mathematics • quadratic functions • circle geometry • the bell curve	• irrational numbers • exponents • polynomial and rational expressions • mathematical expectations • growth patterns • linear and non-linear systems • operations on functions • mathematical reasoning • exponential and logarithmic functions • conics • combinations • trigonometric functions

If you want to transfer from the Applied Mathematics 10–20–30 sequence to the Pure Mathematics 10–20–30 sequence at a future time, you won't have to repeat the topics that are common to pure mathematics and applied mathematics.

If you decide to transfer to Pure Mathematics 20 after successfully completing Applied Mathematics 10, you may have to take a three-credit course called Pure Mathematics 10b. If you decide to transfer to Pure Mathematics 30 after successfully completing Applied Mathematics 20 or Applied Mathematics 30, you may have to take a five-credit course called Pure Mathematics 20b. The two bridging courses are shown in the following diagram.

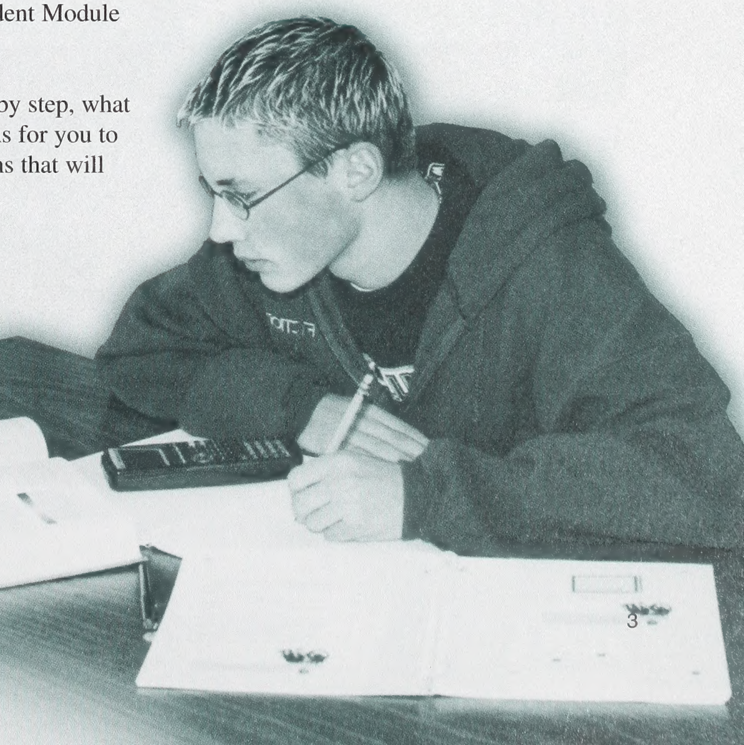


Strategies for Completing Applied Mathematics 30

For each module in Applied Mathematics 30, there is a Student Module Booklet and accompanying Assignment Booklets. The document you are presently reading is called a Student Module Booklet.

Each Student Module Booklet will show you, step by step, what to do and how to do it. There are readings, questions for you to answer in your mathematics binder, and applications that will give you hands-on experience.

It is important to work systematically and carefully through the Student Module Booklets. This work will prepare you for the assignments, projects, and final test.



Following are some suggestions for organizing your mathematics binder:

- Keep a section of your binder to record your responses to the questions in the Student Module Booklet. Also store your marked assignments here.
- Keep a section of your binder for work in progress on your projects. Keep your research notes, plans, rough drafts, and so on.
- Keep a section of your binder to record new skills and concepts, as well as important results and formulas. Get in the habit of describing new skills and concepts in your own words. Record useful ways to help you remember what a concept means. Make charts and diagrams to help you connect mathematical ideas.
- Keep a section of your binder to record mathematical accomplishments. This can include solutions to problems that you are proud of solving. It can also include landmark events, such as when you grasped a difficult concept (an "aha!" experience), or when you used a calculator or spreadsheet in a new way.

Mathematical Processes

Throughout this course, you will be expected to perform the following mathematical processes:

- Connect mathematical ideas to everyday experiences and to concepts in other disciplines.
- Develop and use problem-solving strategies.
- Reason and justify your answers.
- Communicate mathematical ideas.
- Select and use appropriate technologies to solve problems.
- Develop and use estimation and mental-math strategies.
- Use visualization to assist in processing information, making connections, and solving problems.

In order to develop these mathematical processes more fully, you are encouraged to ask someone who is also taking Applied Mathematics 30 to be your study partner. You will find that having a friend to discuss mathematical ideas with will make your studying more enjoyable.

Resources You Will Need

In addition to the course materials for Applied Mathematics 30, you will need the following resources:

- the *Addison-Wesley Applied Mathematics 12 Source Book*, Western Canadian Edition, published by Addison Wesley Longman Ltd. (2002)
- the *Addison-Wesley Applied Mathematics 12 Project Book*, Western Canadian Edition, published by Addison Wesley Longman Ltd. (2002)
- a binder, lined loose-leaf paper, graph paper, dividers, pencils, eraser
- metric and imperial measuring devices, such as a ruler, yardstick, metre-stick, and tape measure
- a mathematical instrument set (compass, protractor, and triangles)
- a computer with a spreadsheet program

  Note: Two popular spreadsheet programs are *ClarisWorks*™ and Microsoft® *Excel*.

- a graphing calculator

  Note: Where it is applicable, the examples in this course and the textbook show the TI-83 calculator; however, all of the graphing calculators in the following chart are approved for use on tests.

Texas Instruments	Sharp	Casio	Hewlett-Packard
TI-83	EL-9600C	Algebra FX 2.0	HP 39g †
TI-83 Plus	EL-9600*	CFX-9850 GA-Plus*	
TI-86	EL-9200*	CFX-9850 G*	
TI-89	EL-9300*	CFX-9800 G*	
TI-92*		FX-9700 series*	
TI-92 Plus			

*no longer commercially available
† The HP 39g calculator will remain on the approved list for the 2001–2003 school years and will then be deleted from the approved list.

If you intend to use the TI-83 or TI-83 Plus graphing calculator, it is recommended that you obtain the video program *The TI-83 Graphing Calculator Video Tutor*.

Many of the resources you will need may be purchased locally or from the Learning Resources Centre (LRC). Following is the LRC website:

http://www.lrc.learning.gov.ab.ca

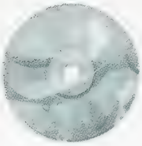
You may wish to discuss the availability of resources with your teacher, as your school division may have a loan policy.

Visual Cues

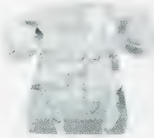
You will find many visual cues in this course. Colour is used to highlight terms that are defined in the Glossary of the Appendix of each Student Module Booklet. You will also find several icons in the margins. Read the following explanations to discover what the various icons prompt you to do.

Refer to the textbook or the Project Book.

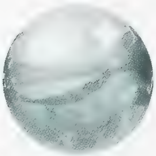
Work with a computer.



Refer to the Applied Mathematics 30 CD.



Contact your teacher for additional information.



Explore the Internet.

Complete specified questions in the Assignment Booklet.

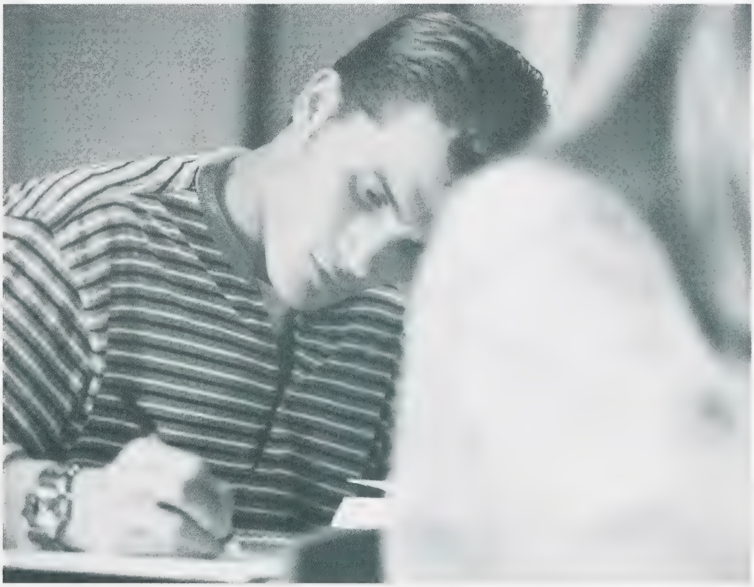
Remember: Any Internet website address given in this module is subject to change.

Where Can I Obtain Diploma Examination Information?

Alberta students will write a diploma examination at the end of the course. Alberta Learning provides several documents to help students prepare for this examination. These documents are found under the heading "Diploma Examinations" at the following Alberta Learning website:

http://www.learning.gov.ab.ca/k_12/testing

Information like course expectations, the makeup of the diploma examination, keyed copies of previous examinations, preparation guides, and calculator policies are available to students at this site.



Each year, in February and September, Alberta Learning provides teachers with information on a **student project**, which teachers **may** use as part of your overall assessment. Information to students will also be posted on the Alberta Learning website. Check with your teacher to determine what you will be expected to do. Be aware that one of the diploma examination's written-response questions will deal with elements of this project and is worth 10% of your diploma examination mark.

You should take advantage of the many sources of information about Applied Mathematics 30. Your success depends on your understanding of course expectations and evaluation procedures. Work closely with your teacher and do not hesitate to ask questions.

Remember, take the initiative to find out all you can about Applied Mathematics 30.

MODULE OVERVIEW

When Orville and Wilbur Wright were at Kitty Hawk in 1904 preparing for their first powered-plane flight, there were no professional pilots to fly their plane. The only way to learn to fly was to build a plane and fly it. How comfortable would you be with boarding a plane for a fun, tropical holiday if you knew that the pilot had just started flying today? Modern airplanes are not the simple machines that Orville and Wilbur worked with, and the preparation for flying one is quite different, too.

To be a jumbo-jet pilot for a major airline, you need thousands of flying hours in progressively more complicated airplanes. You need to master the controls of each new plane before you sit down in the captain's seat, but how can you do that? The flight-training industry uses flight simulators that allow you to learn to fly a plane without any physical danger to yourself or the aircraft. These mechanical wonders tilt, roll, and dive much like the real thing, but you never leave the ground. Like the engineering marvels that they simulate, these machines are operated by powerful computers running sophisticated software to calculate their location and orientation in the air and what you see "out the window."

To create a realistic view of the world as you control the simulator, the graphics processors are busy calculating exactly what to display. A simple flight simulator that you can run on your computer would have to make thousands of vector and matrix calculations each second.

If you are interested in a career in the pilot's seat, you might find the following website interesting:

http://www.edmontonflyingclub.com

In this module you will represent motion using vectors. You will add vectors using scale diagrams and computation. You will also solve two-dimensional and three-dimensional problems involving vectors. You will then use your knowledge and understanding of vectors to analyse traffic collisions and determine how these collisions occurred.

Assessment

Accompanying this Student Module Booklet are two Assignment Booklets. Your grading in this module will be based upon the assignments you submit for assessment. The mark distribution is as follows:

Assignment Booklet 7A	
Activities 1 to 3 Assignment	**70 marks**
Assignment Booklet 7B	
Activities 4 and 5 Assignment	**42 marks**
Module Review Assignment	**43 marks**
Module Project	**40 marks**
TOTAL	**195 marks**

Remember that Activities 1 to 5 in this Student Module Booklet will prepare you for completing the module project and the module assignments. You should work through these activities carefully and compare your answers with the suggested answers provided in the Appendix.

The Module Review provides a review of the module and an enrichment activity. You may choose to do some or all of the questions in the Module Review. Again, you should compare your answers with the suggested answers provided in the Appendix.

Module Project
Accident Reconstruction

Beginning the Project

Your teacher may not require you to complete all the projects provided in this Applied Mathematics 30 distance learning course. Contact your teacher and check whether you need to complete the module project, Accident Reconstruction, as part of your assessment.

Accident scenes often appear to be in total disarray. However, police often are required to reconstruct the scene of an accident to determine what happened. The police especiall need to measure the distance and direction the vehicle travelled after the initial impact. Using vectors and the Law of Conservation of Momentum, police can determine how a collision occurred and the speed of each vehicle.

Accident reconstruction is one of the responsibilities of the RCMP or a local police service, such as the Edmonton Police Service or the Calgary Police Service. Turn to page 302 of the textbook and read "Accident Reconstruction." Complete the questions posed, and store your responses in the project section of your mathematics binder. You may not be able to answer all of the quesions at this time; but as you work through this module, you should return to your mathematics binder and answer these questions in more detail.

After you have recorded some of your initial ideas about accident reconstruction, begin searching for more information about accident reconstruction. You may wish to begin your research by visiting the Internet site described on page 302 of the textbook. This website provides a list of other websites you may find helpful. You may also enter the words *accident reconstruction* into one of the Internet's search engines. Be sure to keep notes about your findings in the project section of your mathematics binder.

As you work through Activities 1 to 5, continue your research on accident reconstruction. Use the Internet, visit your local library, and/or contact your local police station to discover more information as to how automobile accidents are investigated.

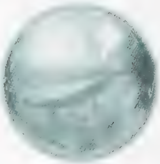
Working through Activities 1 to 5 will help you gain the skills and concepts needed to complete this module project. You will be given more direction on how to complete this project later in this module. In the meantime, feel free to discuss this project with a study partner or a family member. Remember, the work on the project you submit must be your own.

ACTIVITY 1

Vectors and Scalars

Rogaining is one of the fastest growing sports in Australia, Europe, and North America. Rogaining involves hiking to as many checkpoints as possible, over various types of terrain, in a given time period. To get to the various checkpoints, you need to know the distance and direction you have to go. Although you don't follow a straight path from one checkpoint to another, the path can be thought of as a series of distances in a numbe of directions. Distance by itself is known as a **scalar**. A **vector** represents distance and direction.

In this activity you will develop an understanding of scalars and vectors as well as the skills required to construct scale diagrams of vectors.



For more information on rogaining, enter the word *rogaining* into one of the Internet's search engines .

Turn to pages 304 and 305 of the textbook and read the introductory paragraphs of Tutorial 7.1, "Vectors and Scalers."

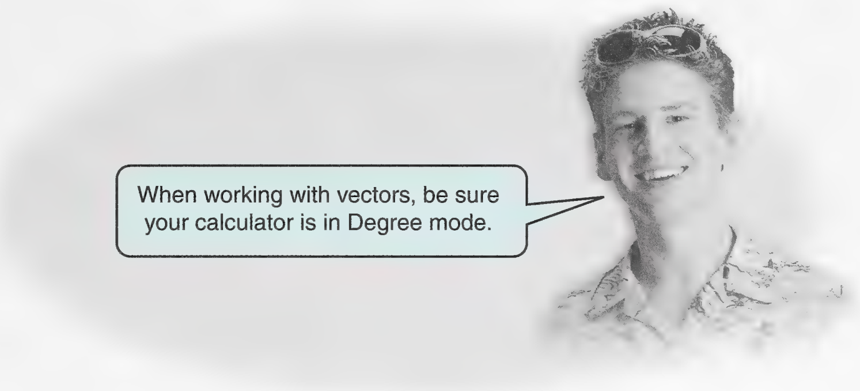
1. What is a geometric vector?

2. In the description of hiking in the second paragraph on page 304, when the lost person first called the ranger and stated that he or she was about 1 km from Turtle Pond, what type of quantity was used?

Compare your responses with the suggested answers in the Appendix, Activity 1, page 52.

Turn to page 1 of Assignment Booklet 7A
and answer questions 1, 2, and 3.

Often, when working with vectors, a reference is made to the term *bearing*. A **bearing** is a three-digit angle measured clockwise from north between the north line and the desired direction. **Note:** A three-digit angle means that angles less than 100° have a leading zero (e.g., 010°, 099°). Refer to page 369 of the Student Reference section of the textbook for illustrated examples.



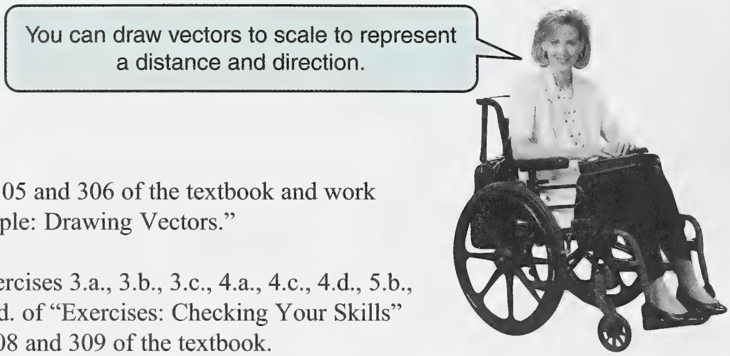
Turn to page 2 of Assignment Booklet 7A
and answer question 4.

3. Draw a diagram of a vector illustrating the following bearings.

a. 050° **b.** 210°

4. Answer exercises 1 and 2 of "Exercises: Checking Your Skills" on page 307 of the textbook.

Compare your responses with the suggested answers in the Appendix, Activity 1, pages 52–53.



Turn to pages 305 and 306 of the textbook and work through “Example: Drawing Vectors.”

5. Answer exercises 3.a., 3.b., 3.c., 4.a., 4.c., 4.d., 5.b., 5.c., and 5.d. of “Exercises: Checking Your Skills” on pages 308 and 309 of the textbook.

Compare your responses with the suggested answers in the Appendix, Activity 1, page 53.

Vectors can be drawn equal in magnitude and in the same direction or in opposite directions. Such vectors are parallel to each other and often are used to represent something that is done in formation. Turn to pages 306 and 307 and read the information about equal and opposite vectors following the example. Pay particular attention to the definitions of equal and opposite vectors in the coloured box on page 307.

6. Answer exercises 6, 7, and 8 of “Exercises: Checking Your Skills” on page 309 of the textbook.

Compare your responses with the suggested answers in the Appendix, Activity 1, page 54.

Turn to page 2 of Assignment Booklet 7A and answer questions 5 and 6.

Looking Back

In this activity you developed an understanding of scalars and vectors. You also learned to label vectors and to construct scale diagrams to represent vectors.

7. Turn to page 309 of the textbook and answer “Communicating the Ideas.”

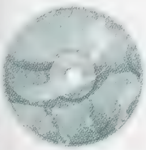
Compare your response with the suggested answer in the Appendix, Activity 1, page 55.

ACTIVITY 2

Adding Vectors Using Scale Diagrams

Have you ever gone on a treasure hunt in which you had to locate various items by following directions? You may have had to walk 35 paces southeast from the starting position to the edge of the flowerbed, then 23 paces south toward the lilac bush, and so on. These distances and directions can be represented by vectors. In this activity you will explore ways to add vectors.

To combine two vectors, you need to know the length and the direction of each vector. You will begin your study of vector addition by combining several vectors of given lengths and directions.

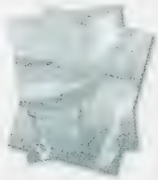


As a quick introduction to adding vectors, view the segment *Vectors and Navigation* on the Applied Mathematics 30 CD.

Turn to page 312 of the textbook and read "Investigation 1: Adding Vectors That Represent Changes in Position." Construct a square to represent the diagram; then construct each required vector. An appropriate scale is $1\ \text{cm} = 10$ paces. You may construct your diagram by hand on plain paper or grid paper, or you may use *The Geometer's Sketchpad*® to do the construction.

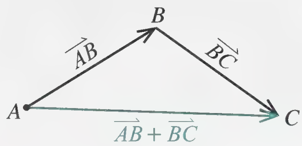
1. Complete exercises 1, 2, and 3 of "Investigation 1: Adding Vectors That Represent Changes in Position" on page 312 of the textbook.

Compare your responses with the suggested answers in the Appendix, Activity 2, pages 55–56.



Turn to page 3 of Assignment Booklet 7A and answer question 7.

Two vectors can be added by placing them head-to-tail or tail-to-tail. When adding vectors head-to-tail, the vector representing the sum is the vector that can be drawn from the tail of the first vector to the head of the second vector. This vector is called the **resultant**.



The **triangle method of adding vectors** can be used to solve problems involving the addition of vectors that are drawn tail-to-head. Study the following example.

Example

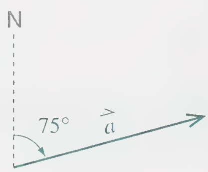
A submarine travels at 15 knots on a bearing of 075° for 1 h; then it turns to a bearing of 130° and travels for an additional 2 h. Determine the displacement of the submarine from the original starting point.

Solution

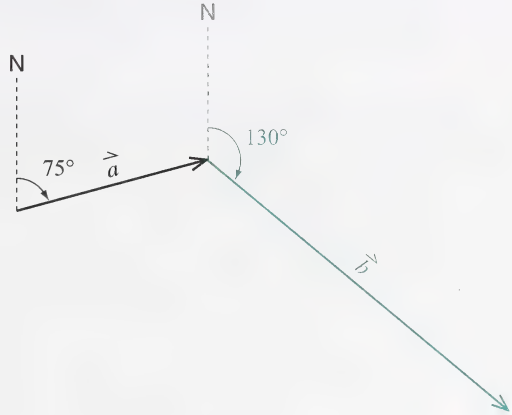
Note: 15 knots = 15 nautical miles/hour

Step 1: Decide on a convenient scale. Since the submarine is travelling for 1 h and then for 2 h, it will travel 15 nautical miles in the first direction and 30 nautical miles in the second direction. Therefore, a scale of 1 cm = 5 nautical miles would be convenient.

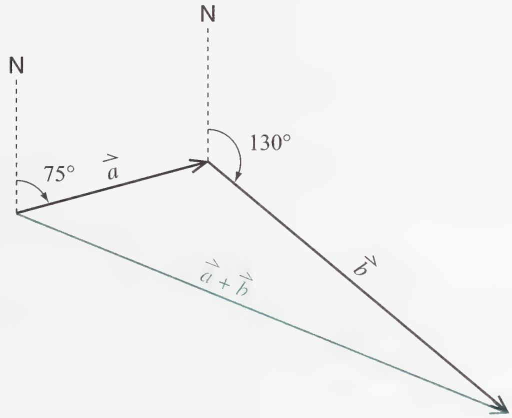
Step 2: Draw a diagram, starting with the north line and the vector representing the first leg of the trip. Use a protractor to find a bearing of 075° and draw a line that is 3 cm long.



Step 3: Draw another north line at the tip of this vector and use a protractor to find a bearing of 130°. Draw a line that is 6 cm long from the tip of the first vector.



Step 4: Draw in the resultant vector from the initial starting point to the tip of the second vector.



Step 5: Measure the length of the resultant vector and its bearing.

$$\begin{aligned}\text{Length of } \vec{a}+\vec{b} &= 8.1 \text{ cm} \times 5 \text{ nautical miles/cm} \\ &= 40.5 \text{ nautical miles}\end{aligned}$$

$$\text{Bearing } = 112°$$

The displacement of the submarine is 40.5 nautical miles $[112°]$.

2. Turn to page 318 of the textbook and answer exercises 1, 2, and 3 of "Exercises: Checking Your Skills."

Compare your responses with the suggested answers in the Appendix, Activity 2, pages 56–57.

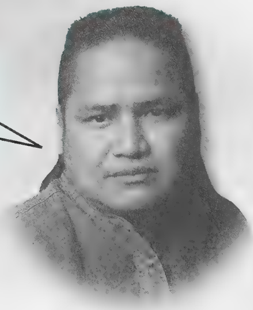
The method of adding vectors you just used is the triangle method. Another method of adding vectors is the **parallelogram method**. The parallelogram method is used when the vectors start from the same point. For more information on methods for adding vectors, turn to page 313 of the textbook and read the paragraphs preceding Investigation 2.

3. Turn to page 313 of the textbook and complete exercises 1 and 2 of "Investigation 2: The Parallelogram Method for Adding Two Vectors."

Compare your responses with the suggested answers in the Appendix, Activity 2, pages 57–58.

Turn to page 3 of Assignment Booklet 7A and answer question 8.

You have learned that vectors can be added using one of two methods. Turn to page 314 of the textbook and read the bulleted items in the coloured box.



4. Answer exercises 1, 2, and 3 of "Discussing the Ideas" on page 317 of the textbook.

Compare your responses with the suggested answers in the Appendix, Activity 2, page 58.

A significant part of the study of physics is about forces. A force is a push or a pull exerted on an object. Forces have both magnitude and direction and, therefore, are represented by vectors.

Turn to page 314 of the textbook and read the paragraph following the coloured box. Then read “Investigation 3: A Study of Forces,” including exercises 1 to 8 on pages 314 and 315, to get an overview of how the investigation is to be completed. If you have access to the equipment from a laboratory, you may complete exercises 1 to 5 yourself. If you do not have access to the required equipment, review the following tables.

Exercise 3

Mass (g)	Force on First Scale (N)	Force on Second Scale (N)	Measure of θ (degrees)
300	1.9	1.9	75
400	2.5	2.5	75
500	3.2	3.2	75

Exercise 4

Mass (g)	Force on First Scale (N)	Force on Second Scale (N)	Measure of θ (degrees)
300	2.1	2.1	90
400	2.6	2.6	90
500	3.4	3.4	90

Exercise 5

Mass (g)	Force on First Scale (N)	Force on Second Scale (N)	Measure of θ (degrees)
300	1.8	1.8	68
400	2.4	2.4	68
500	3.0	3.0	68

5. Complete exercises 6, 7, and 8 of “Investigation 3: A Study of Forces” on page 315 of the textbook. For the comparison in exercise 6, you may wish to make a table comparing the resultant force for each mass at each angle.

Compare your responses with the suggested answers in the Appendix, Activity 2, pages 58–61.

Resultant forces are part of many problems in physics and science. The vectors that represent the initial forces often have to be added together in a geometric figure.

For more information on resultant forces and adding forces in geometric figures, turn to pages 316 and 317 and work through "Example 1: Determine a Resultant Force" and "Example 2: Adding Vectors in a Geometric Figure."

6. The equilibrant is defined as the force acting on an object in the opposite direction to the resultant force. Use this definition to draw the equilibrant in the diagram in Example 1.

7. Answer the following exercises on pages 318 and 319 of the textbook.

a. exercises 6, 7, and 8 of "Exercises: Checking Your Skills"

b. exercise 10 of "Exercises: Extending Your Thinking"

Note: Answers to the exercises at the back of the textbook may not be accurate.

Compare your responses with the suggested answers in the Appendix, Activity 2, pages 61–63.

Turn to pages 4 and 5 of Assignment Booklet 7A and answer questions 9 and 10.

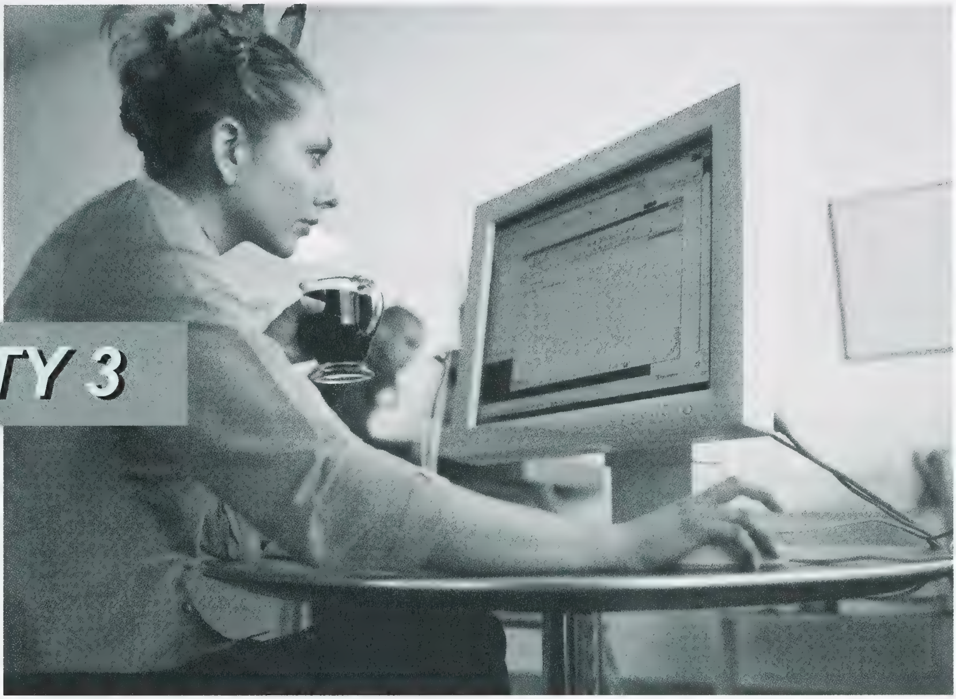
Looking Back

To see some examples of vectors being used in engineering, view the segment *Vectors and Supporting Structures* on the Applied Mathematics 30 CD.

In this activity you used two methods—the triangle method and the parallelogram method—to add vectors using scale diagrams. You then solved vector problems involving forces for magnitude and direction of the resultant using scale diagrams.

8. Turn to page 319 of the textbook and answer "Communicating the Ideas."

Compare your response with the suggested answer in the Appendix, Activity 2, page 64.



ACTIVITY 3

Multiplying a Vector by a Scalar

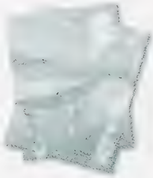
Have you used computer programs that let you increase and decrease the size of photographs or other graphics? Such programs use vectors and scalar multiplication in the programming to increase and decrease the size of the images on the screen.

In this activity you will multiply vectors by scalars.

Recall that the multiplication of numbers can be thought of as repeated addition. Similarly, repeated vector addition can be thought of as the multiplication of a single vector by the number of times it is added to itself. The number that the vector is multiplied by is a scalar. That is, it has magnitude or size; but, unlike vectors, it has no direction.

1. Complete exercises 1 to 9 of "Investigation 1: Scalar Multiples of a Vector" on page 320 of the textbook.

Compare your responses with the suggested answers in the Appendix, Activity 3, pages 64–65.

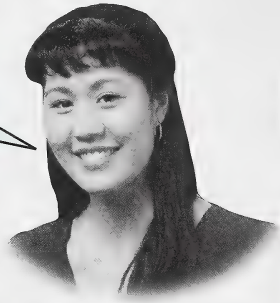


Turn to page 6 of Assignment Booklet 7A and answer question 11.

Turn to page 320 of the textbook and read the summary of vector multiplication by a scalar in the coloured box.

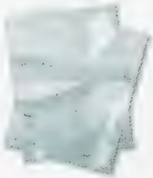
2. Are the summary statements in the coloured box representative of what you stated in your summary? What additional statement is made in the coloured box?

As mentioned in the introduction to this activity, scalar multiples are commonly used in the enlargement and reduction of figures in computer graphics.



3. Complete exercises 1 to 6 of "Investigation 2: Enlarging Figures" on page 321 of the textbook.

Compare your responses with the suggested answers in the Appendix, Activity 3, pages 65–66.



Turn to pages 6 and 7 of Assignment Booklet 7A and answer question 12.

Now, you will use scalar multiplication to solve problems.

REMEMBER

- *To increase the size of a vector, multiply the vector by a scalar greater than 1.*
- *To decrease the size of a vector, multiply the vector by a scalar between zero and 1 (a fraction).*
- *To reverse the direction of a vector, multiply the vector by a negative scalar.*

Turn to pages 321 and 322 of the textbook and work through “Example: Scale Drawings of a Scalar Multiple of a Vector.”

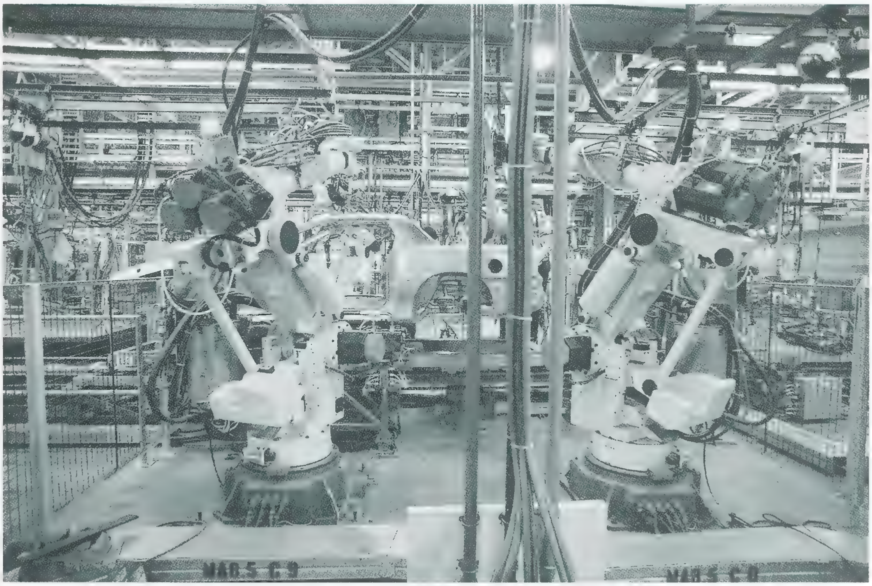
4. Answer the following exercises on pages 322 and 323 of the textbook.

- **a.** exercises 2, 3, and 4 of “Exercises: Checking Your Skills”
- **b.** exercise 5.a. and 5.c. of “Exercises: Extending Your Thinking”

Compare your responses with the suggested answers in the Appendix, Activity 3, pages 67–70.

Turn to pages 7 and 8 of Assignment Booklet 7A and answer questions 13, 14, and 15.

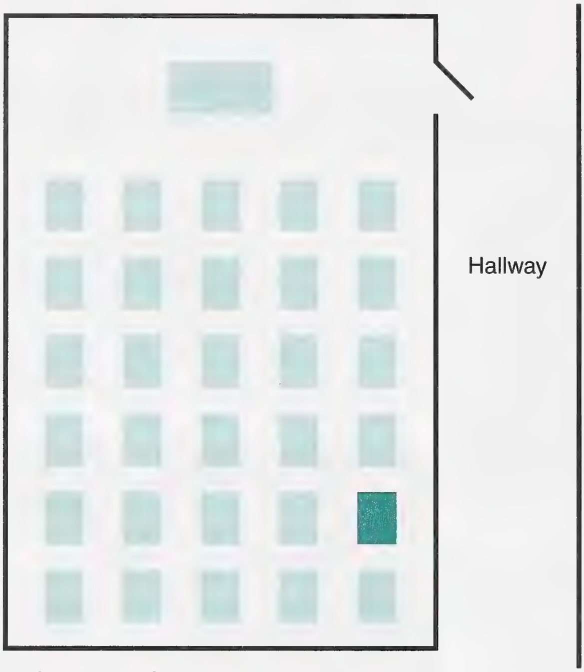
Project: Training a Robot



Now you will complete a project from the *Addison-Wesley Applied Mathematics 12 Project Book* that involves the use of vectors and scalar multiplication. Turn to page 130 of the Project Book and read “The Task” and “Background.” Pay particular attention to the bulleted questions at the bottom of the page.

5. How are vectors, directions, and robots related?

6. Turn to page 132 of the Project Book and read exercises 1 to 3 of "Getting Started." Use the classroom plan given and the selected desk to answer the questions that follow.



scale: 1 cm = 5 feet

   a. Draw the shortest path from the selected desk to the opposite wall in the hallw outside the door of the classroom.

   b. How many straight-line paths does it take?

   c. State the distance of the path using the given scale.

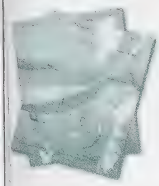
7. Choose a desk on the opposite side of the classroom. In terms of direction, how would a pathway from that desk to the opposite wall in the hallway outside the door of the classroom compare to the pathway in exercise 6?

8. What type of convention could you use to describe the pathway and directions in exercises 6 and 7?

9. What is the purpose of establishing a convention for direction?

10. Complete exercises 4 and 5 of "Getting Started" on pages 132 and 133 of the Project Book. Trace templates A and B on page 135. For exercise 5, divide the magnitude of each vector by 1.5 because the template you trace will be too small for the given magnitudes. Round each magnitude to the nearest tenth, and use the same direction.

Compare your responses with the suggested answers in the Appendix, Activity 3, pages 70–73.



Turn to pages 8 to 10 of Assignment Booklet 7A and answer questions 16, 17, and 18.

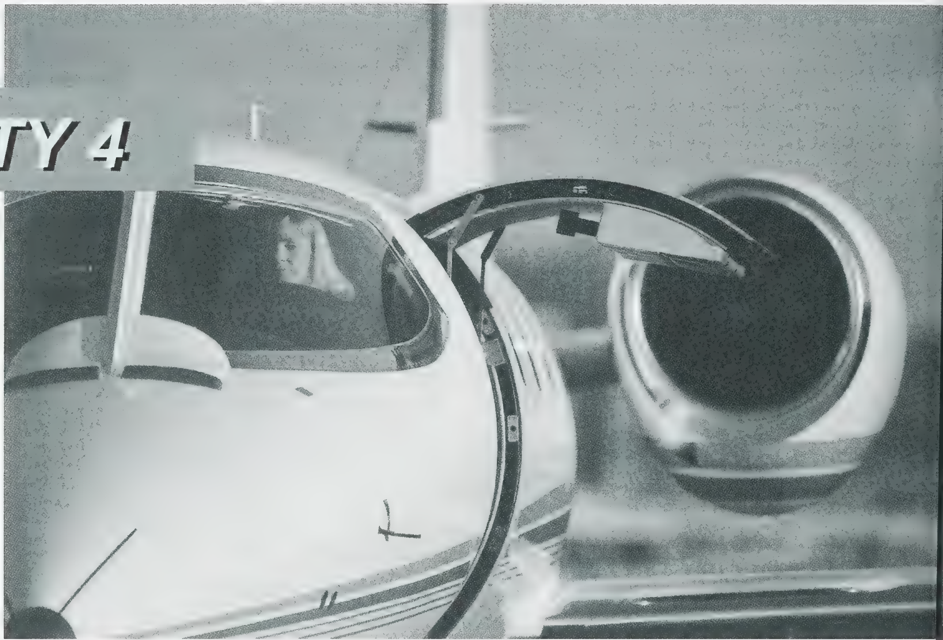
Looking Back

In this activity you multiplied a vector by a scalar and found that such multiplication is the same as adding the vector the number of times indicated by the multiplication.

11. Turn to page 323 of the textbook and answer "Communicating the Ideas."

Compare your response with the suggested answer in the Appendix, Activity 3, page 73.

ACTIVITY 4



Solving Vector Problems by Computation

Although pilots of small aircraft can estimate their speed and direction, onboard instruments provide the pilot with much more accurate information about the speed and direction of the airplane. Similarly, although vector problems can be solved with scale diagrams, this provides only an approximate solution.

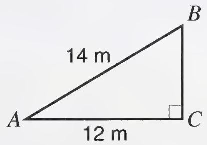
In previous mathematics courses, you used trigonometry to solve problems involving triangles with greater accuracy than you were able to do with scale diagrams.

In this activity you will use trigonometry to solve vector problems with greater accurac

In earlier courses you studied the Pythagorean Theorem, the Sine Law, and the Cosine Law. They are often useful in solving questions involving the addition of vectors. The next three examples will help you to remember how to apply them.

Example 1

What is the length of the shortest side of triangle *ABC*? Round your answer to 1 decimal place.



Solution

Because triangle *ABC* is a right-angle triangle, the Pythagorean Theorem can be applied.

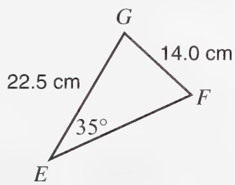
$b = 12$ and $c = 14$

$$\therefore\ c^2 = a^2 + b^2$$
$$(14)^2 = a^2 + (12)^2$$
$$196 = a^2 + 144$$
$$a^2 = 196 - 144$$
$$a^2 = 52$$
$$a \doteq 7.2$$

The length of the shortest side of triangle *ABC* is approximately 7.2 m.

Example 2

What is the measure of $\angle F$ in the following triangle? Round your answer to the nearest degree.

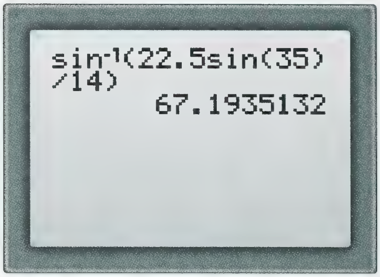


Solution

Use the Sine Law to determine $\angle F$.

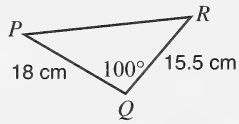
Sine Law : $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$$\begin{aligned}
\frac{e}{\sin E} &= \frac{f}{\sin F} \\
\frac{14}{\sin 35^\circ} &= \frac{22.5}{\sin F} \\
14(\sin F) &= 22.5(\sin 35^\circ) \\
\sin F &= \frac{22.5 \sin 35^\circ}{14} \\
\angle F &= \sin^{-1}\left(\frac{22.5 \sin 35^\circ}{14}\right) \\
&\doteq 67^\circ
\end{aligned}$$



Example 3

What is the length of side PR in the following triangle? Round your answer to the nearest tenth.



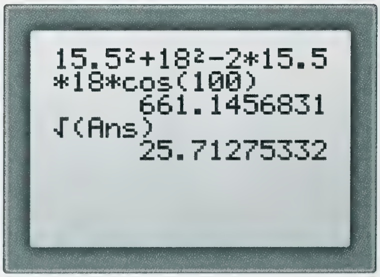
Solution

Use the Cosine Law to determine side PR.

Cosine Law : $c^2 = a^2 + b^2 - 2ab\cos C$

$p = 15.5$, $r = 18$, $\angle Q = 100^\circ$, and $q = PR$

$$\begin{aligned}
\therefore\ q^2 &= p^2 + r^2 - 2pr\cos Q \\
q^2 &= (15.5)^2 + (18)^2 - 2(15.5)(18)\cos 100^\circ \\
q^2 &\doteq 661.145\,683\,2 \\
q &\doteq 25.7
\end{aligned}$$



Side PR is approximately 25.7 cm in length.

To practise your previous skills involving the Pythagorean Theorem, Sine Law, and Cosine Law, complete the following exercises. If you need help, refer to the examples and definitions in the Student Reference Section of the textbook on pages 367 to 391.

1. Answer exercises 1, 2, and 4 of "Practise Your Prior Skills" on page 324 of the textbook.

Compare your responses with the suggested answers in the Appendix, Activity 4, pages 73–74.

Turn to pages 1 and 2 of Assignment Booklet 7B and answer question 1.

Most problems involving vectors result in the formation of triangles. Therefore, you can use trigonometric methods to solve vector problems just as you used them to solve similar problems involving triangles.

When solving vector problems, find the magnitude to the nearest tenth and the angle to the nearest degree unless other instructions are given.

Turn to page 325 of the textbook and read the five steps for solving vector problems using trigonometry. Then work through "Example 1: Use Right Angle Trigonometry and the Pythagorean Theorem" and find the five steps for solving a vector problem using trigonometry. **Note:** Often a question will state "Determine the resultant velocity." To determine the resultant velocity implies that you determine both the magnitude and direction of the final velocity.

2. Answer the following exercises on pages 328 and 329 of the textbook.

a. exercises 1, 2, and 6 of "Exercises: Checking Your Skills"
b. exercise 11 of "Exercises: Extending Your Thinking"

Compare your responses with the suggested answers in the Appendix, Activity 4, pages 75–77.

Turn to pages 2 and 3 of Assignment Booklet 7B and answer question 2.

In the preceding exercises you solved vector problems involving vectors at right angles only. Many vector problems involve angles other than right angles. These types of problems require the use of the Sine Law and Cosine Law.

Turn to pages 326 to 328 of the textbook and work through "Example 2: Use the Sine Law and the Cosine Law." Pay particular attention to how the required angles are determined.

3. Why is the Cosine Law used to calculate the resultant in Example 2?

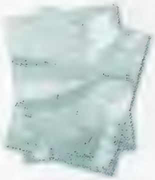
4. Why is $\angle BAD$ equal to 115°?

5. Explain why $\angle ADC$ is equal to 65°.

6. Why is the bearing of the resultant vector equal to 330° plus 15.2°?

7. Answer exercises 4 and 8 of "Exercises: Checking Your Skills" on pages 328 and 329 of the textbook.

Compare your responses with the suggested answers in the Appendix, Activity 4, pages 78–80.



Turn to pages 4 and 5 of Assignment Booklet 7B and answer questions 3 and 4.

Looking Back

In this activity you applied your prior knowledge of trigonometry to solve vector problems. You used right-angle trigonometry and the Pythagorean Theorem to solve vector problems that involved right angles. You then used the Sine Law and Cosine Law to solve vector problems that consisted of no right angles.

Two-dimensional vector problems require solutions with two parts: a magnitude and direction. In the next activity you will work with three-dimensional vector problems.

8. Turn to page 329 and answer "Communicating the Ideas."

Compare your response with the suggested answer in the Appendix, Activity 4, page 80–82.

ACTIVITY 5

Vector Problems in Three Dimensions

Jacques is an air-traffic controller at the Edmonton International Airport. He watches the radar screen for incoming and outgoing airplanes. The direction and speed of the airplanes are represented by moving dots on the radar screen. The motion of these dots can be represented by a vector. Vectors represent magnitude and direction of a moving object.

The direction of departing and landing aircraft must be seen in three dimensions since the aircraft are not only moving forward in a horizontal plane—they are also ascending or descending.

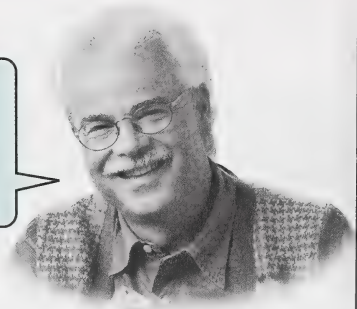
In this activity you will solve simple three-dimensional vector problems.

Turn to page 330 of the textbook and read the introductory paragraphs of Tutorial 7.5, "Vector Problems in Three Dimensions."

1. Two-dimensional problems can be illustrated by drawing vectors directly on a sheet of paper. Speculate how you would represent a three-dimensional diagram on a flat sheet of paper.

Compare your response with the suggested answer in the Appendix, Activity 5, page 83.

A three-dimensional object, such as a shoe box, can be very helpful when visualizing three-dimensional drawings. In the next exercise you will use a shoe box to help you visualize the displacement of three-dimensional vectors.



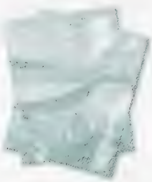
Turn to pages 330 and 331 of the textbook and read "Investigation 1: Displacements in Three Dimensions."

2. Complete exercises 1 to 5 of "Investigation 1: Displacements in Three Dimensions." In your summary in exercise 5, determine the direction of $\vec{d}_{12}$ and $\vec{d}$.

Compare your responses with the suggested answers in the Appendix, Activity 5, pages 83–84.

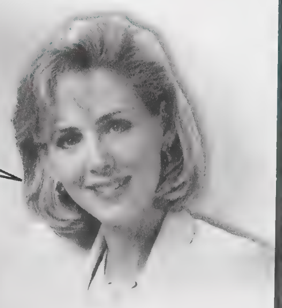
You should have found the shoe box useful in visualizing a path of a three-dimensional vector. Sketches of three-dimensional vector problems are usually done as if they occur in a transparent box.

Turn to page 331 of the textbook and read the information following Investigation 1.

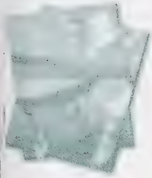


Turn to pages 5 and 6 of Assignment Booklet 7B and answer question 5.

Now that you have had some practice sketching 3-D box diagrams, you are ready to solve some problems involving vectors in three dimensions.



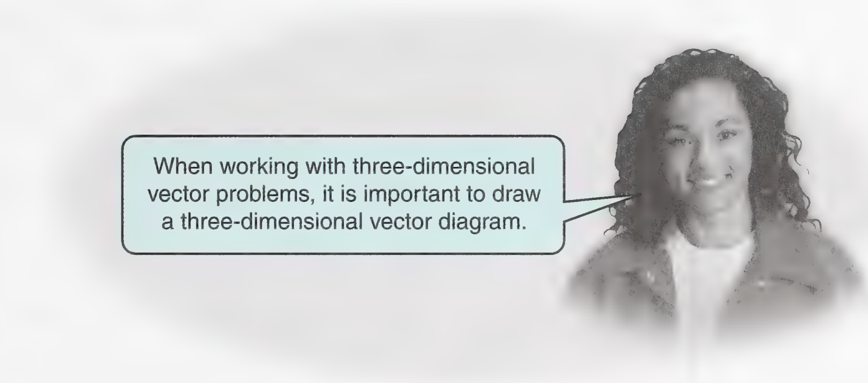
Turn to pages 332 and 333 of the textbook and work through "Example 1: Combine Three Forces, Two of Which Are Parallel" and "Example 2: Three Perpendicular Forces."



Turn to page 7 of Assignment Booklet 7B and answer questions 6 and 7.

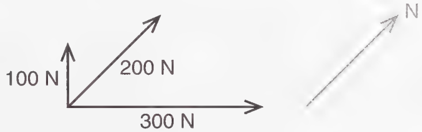
3. Answer exercises 1 and 2 of "Exercises: Checking Your Skills" on page 336 of the textbook.

Compare your responses with the suggested answers in the Appendix, Activity 5, pages 85–86.



Example

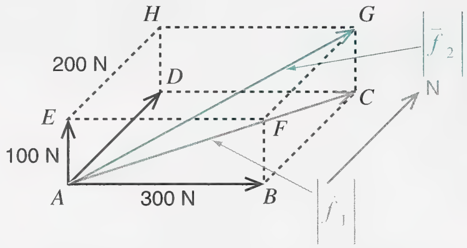
Three mutually perpendicular forces are acting on an object (as shown in the diagram given). Determine the resultant force.



Solution

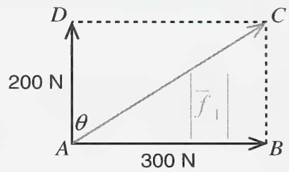
To find the resultant force, you need to determine the magnitude of the final resultant vector as well as its direction. The direction is measured both from north (bearing) and from the horizontal.

Step 1: Complete the box. Draw the resultant vectors in the appropriate locations. The orientation of the box is not that important. What is important is the location of the vectors and the labelling of the directions.



Step 2: Determine the resultant force $\left|\vec{f}_1\right|$.

Top View



$$\left|\vec{f}_1\right|^2 = (200)^2 + (300)^2$$

$$\left|\vec{f}_1\right|^2 = 130\ 000$$

$$\left|\vec{f}_1\right| = \sqrt{130\ 000}$$

$$\doteq 360.555\ 127\ 5$$

$$\therefore \left|\vec{f}_1\right| \doteq 360.6\ \text{N}[056°]$$

$$\tan \angle CAD = \frac{300}{200}$$

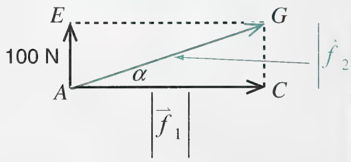
$$\angle CAD = \tan^{-1}\left(\frac{300}{200}\right)$$

$$\doteq 56.309\ 932\ 47°$$

$$\doteq 56°$$

Step 3: Determine the resultant force $\left|\vec{f}_2\right|$.

Diagonal View



$$\left|\vec{f}_2\right|^2 = (100)^2 + \left|\vec{f}_1\right|^2$$

$$\left|\vec{f}_2\right|^2 = 10\ 000 + 130\ 000$$

$$\left|\vec{f}_2\right|^2 = 140\ 000$$

$$\left|\vec{f}_2\right| = \sqrt{140\ 000}$$

$$\doteq = 374.2\ \text{N}$$

$$\tan \angle CAG = \frac{100}{\left|\vec{f}_1\right|}$$

$$\tan \angle CAG = \frac{100}{\sqrt{130\ 000}}$$

$$\angle CAG = \tan^{-1}\left(\frac{100}{\sqrt{130\ 000}}\right)$$

$$\doteq 15.501\ 359\ 57^\circ$$

$$\doteq 16^\circ$$

Step 4: Write a concluding statement.

The resultant force is about 374.2 N, bearing 56° east of north and 16° up.

Remember: Three-dimensional vector problems require solutions with three parts: a magnitude, an angle to the horizontal, and a bearing.

Now, work through exercise 4.

4. A helicopter is traveling 60 m/s [south]. It is descending at an angle of 40° to the horizontal and encounters a wind blowing toward the west at a speed of 20 m/s.

a. Draw a box, label the corners, and enter the given vector information. Draw the resultant vectors in the appropriate locations.

b. Determine the magnitude of the resultant velocity.

c. Determine the angle at which the helicopter is dropping (the angle the resultant makes with the horizontal).

d. Determine the bearing of the helicopter.

e. Write a summary statement about the resultant force.

Compare your responses with the suggested answers in the Appendix, Activity 5, pages 86–88.

Turn to page 8 of Assignment Booklet 7B and answer question 8.

Looking Back

In this activity you developed an understanding of vectors in three dimensions, and you used your knowledge to solve vector problems involving three dimensions.

5. Turn to page 337 of the textbook and answer "Communicating the Ideas."

Compare your response with the suggested answer in the Appendix, Activity 5, page 88.

Module Review

This module dealt with Chapter 7: Vectors in the *Addison-Wesley Applied Mathematics 12 Source Book.*

Turn to page 340 of the textbook and review the skills and concepts that were develope in this module. Also, read the column of important results and formulas you discovere

Answer exercises 2.b., 2.d., 4, 6, and 7 of Part B of "What Should I Be Able to Do?" o pages 341 and 342 of the textbook.

Compare your responses with the suggested answers in the Appendix, Module Review, pages 89–92.

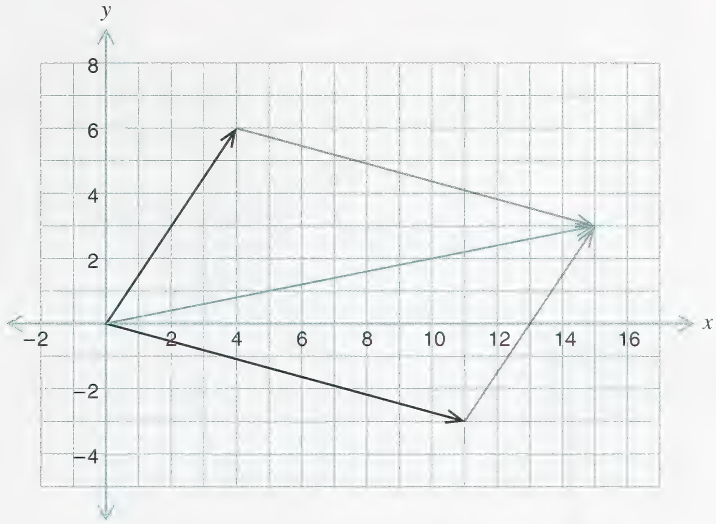
Turn to pages 9 to 20 of Assignment Booklet 7B and complete the Module Review Assignment.

If you had difficulties understanding the skills and concepts in Module 7: Vectors, it i recommended that you contact your teacher for some extra help activities. If you have clear understanding of the skills and concepts in this module, you may wish to do the following enrichment activity. You may decide to do both.

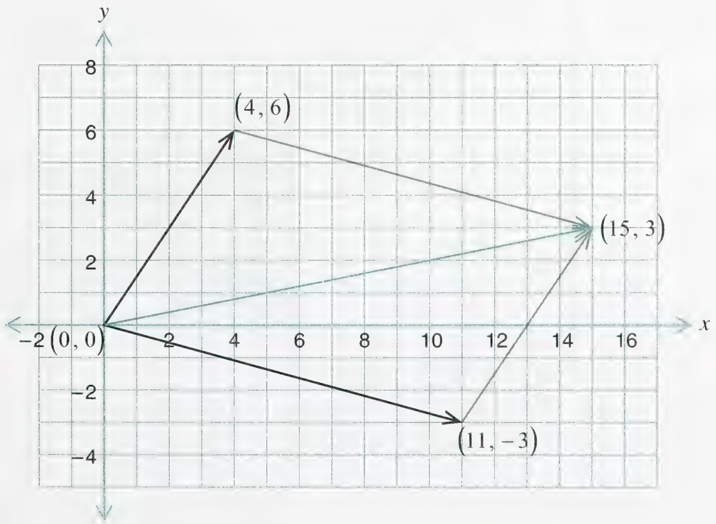
Enrichment

In this module you have been using length and direction to represent a vector. There are other ways to represent vectors that work equally well, and in some cases make the solutions of problems simpler. One of these ways is to use an ordered pair to represent a vector on a plane.

In the following diagram, two vectors are added using the parallelogram method.

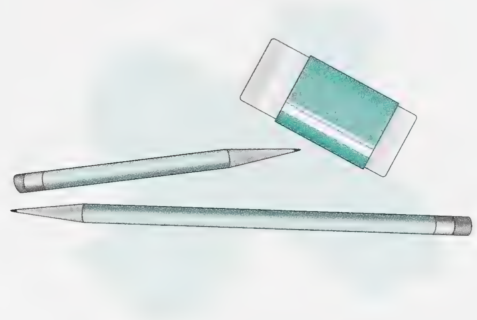


The next diagram shows the coordinates of the heads and tails of the vectors in the previous addition.



Notice that the coordinates of the head of the vector sum, $(15, 3)$, are the sums of the coordinates of the heads of the two added vectors $[4+11 \text{ and } 6+(-3)]$. If all vectors just started at the origin, this would make adding them very easy.

All pairs of vectors can be added using their coordinates. You just have to find the equivalent vectors starting at the origin. You do this by subtracting the tail coordinates from the head coordinates.



Example

What vectors, starting at the origin, have the same length and direction as these vectors?

a. a vector going from $(3, 5)$ to $(11, -2)$

b. a vector going from $(-4, -3)$ to $(1, 3)$

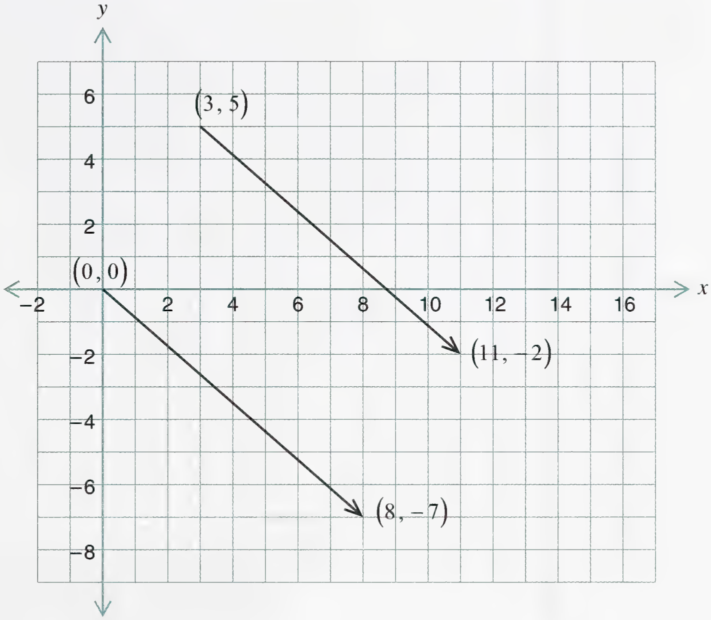
c. a vector going from $(1035.45, 3487.23)$ to $(1091.62, 3452.58)$

Solution

a. The equivalent vector is found by subtracting the tail coordinates from the head coordinates.

	Head	Tail
x-coordinates	11	3
y-coordinates	−2	5

The differences are $11-3=8$ and $-2-5=-7$. The equivalent vector's head is at $(8,-7)$. The initial vector and the equivalent vector starting at the origin are shown in the following diagram.

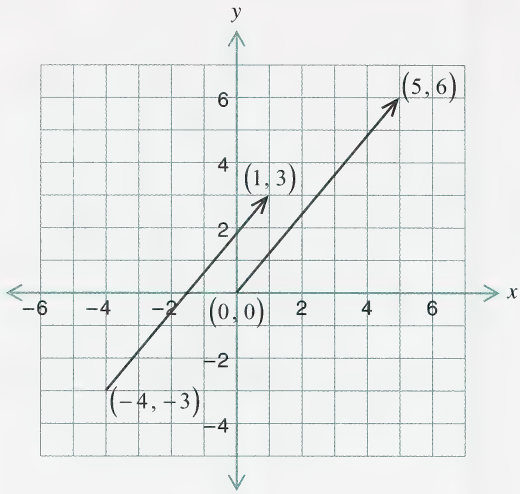


b. The equivalent vector is found by subtracting the tail coordinates from the head coordinates.

	Head	Tail
x-coordinates	1	−4
y-coordinates	3	−3

The differences are $1-(-4)=5$ and $3-(-3)=6$. The equivalent vector's head is at $(5,6)$.

The initial vector and the equivalent vector starting at the origin are shown in the following diagram.



c. The equivalent vector is found by subtracting the tail coordinates from the head coordinates.

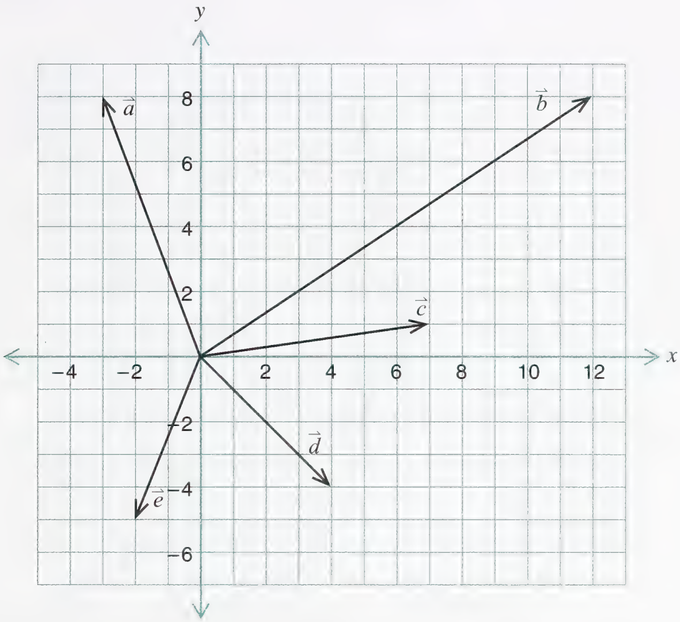
	Head	Tail
x-coordinates	1091.62	1035.45
y-coordinates	3452.58	3487.23

The differences are $1091.62 - 1035.45 = 56.17$ and $3452.58 - 3487.23 = -34.65$.

The equivalent vector's head is at $(56.17, -34.65)$.

Using the ordered pair for the head of an equivalent vector starting at the origin gives a single way to represent equivalent vectors. To differentiate the vector meaning from an ordinary ordered pair, vectors are often written using brackets instead of parentheses. T three vectors from the preceeding example would be writen as $[8, -7]$, $[5, 6]$, and $[56.17, -34.65]$.

1. For each vector in the following diagram, write its bracketed ordered pair.



2. In the diagram in exercise 1, what vector would correspond to these vector sums and differences? Give your answers as bracketed ordered pairs.

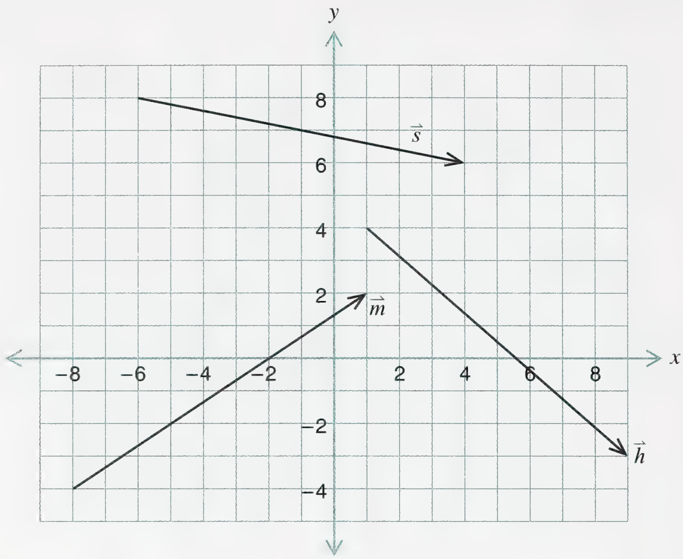
   a. $\vec{a} + \vec{d}$

   b. $\vec{c} + \vec{e}$

   c. $\vec{b} - \vec{d}$

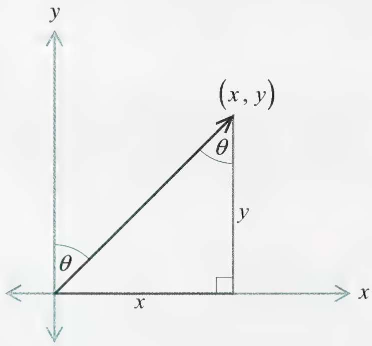
Compare your responses with the suggested answers in the Appendix, Module Review: Enrichment, pages 92–93.

3. For each vector in the following diagram, give its bracketed ordered pair.

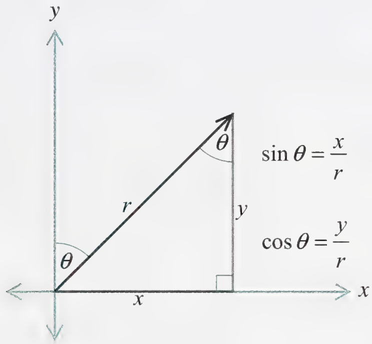


Compare your response with the suggested answer in the Appendix, Module Review: Enrichment, page 93.

If you have a vector given as $[x, y]$, and you need to tell someone what magnitude it and its direction, it's not hard to do. The following diagram shows that the length of th vector is $\sqrt{x^2 + y^2}$ and the required angle is $\theta = \tan^{-1}\left(\frac{x}{y}\right)$.



Getting the ordered pair for a vector showing magnitude and direction is also not difficult. The following diagram shows that the x-component will be $r \times \sin\theta$ and the y-component will be $r \times \cos\theta$. The ordered pair would be $[r \times \sin\theta, r \times \cos\theta]$.



4. What is the magnitude and direction of each of the following vectors?

a. $[3, 4]$

b. $[4, -1]$

5. What is the bracketed form of each of the following vectors?

a. $8[060°]$

b. $3.25[325°]$

6. What is the sum of the vectors $[1, 3]$, $[-7, 4]$, $[8, -7]$, and $7[030°]$? Give the sum in both bracketed form and magnitude-direction form. Round your answers to 2 decimal places where necessary.

Compare your responses with the suggested answers in the Appendix, Module Review: Enrichment, pages 93–95.

Completing the Project

By now you should have completed your initial research for your module project, Accident Reconstruction. You are to recreate the scene of an accident and determine th velocity and direction of travel of each of the vehicles involved. Before you recreate th scene of an accident, you need some understanding of the physics of collisions. You wi begin by simulating an accident using checkers (or some other suitable objects that wil slide on paper). Loonies or toonies will also work quite well for this simulation. This simulation will help you gain an understanding of how objects react after a collision.

Turn to pages 310 and 311 of the textbook and read "Simulating an Accident," includi steps 1 to 6.

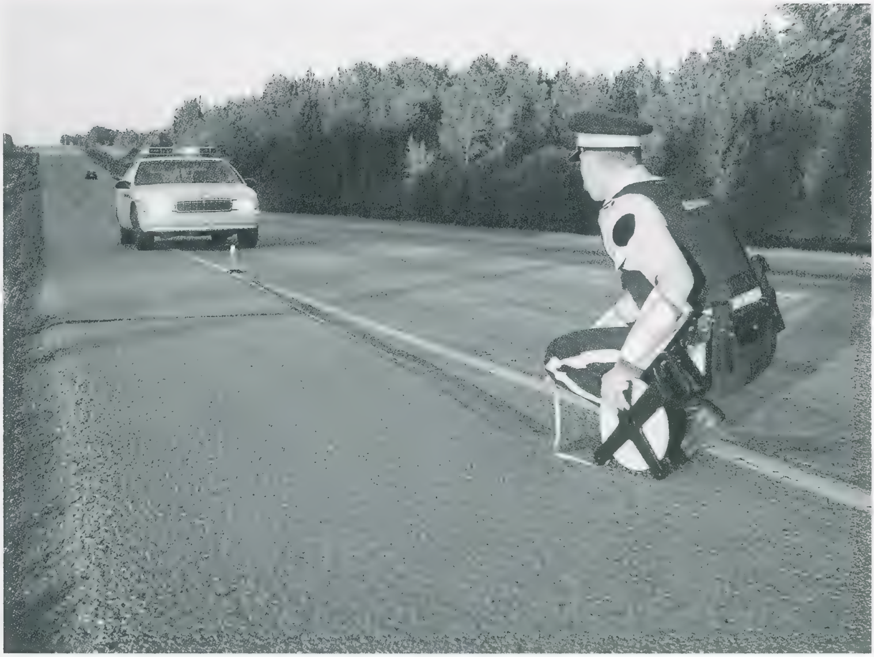
Collect the required materials—a large piece of paper and two objects, such as checker or loonies. Tape two checkers or loonies together to make the heavier object. You may use a length of paper used for covering tables or a large piece of cardboard.

1. Perform the simulation as described in exercises 1 to 6 on pages 310 and 311. It is not necessary to have a partner to do this simulation. Keep all diagrams that you make and make notes on how this simulation relates to an actual accident situation You will need your diagrams for the exercises on page 338 of the textbook.

2. Answer exercise 10 of Part C of "What Should I Be Able to Do?" on page 343 of the textbook.

3. Use your diagrams from the simulation on pages 310 and 311 of the textbook to answer exercises 1 to 4 of "How Fast Was That Car Going?" on page 338.

Compare your response with the suggested answer in the Appendix, Module Project, pages 95–98.



Module Project

Now that you have more insight into the module project, revise your answers. It is now time to complete the module project, Accident Reconstruction.

You may use your responses from the textbook exercises on pages 302, 310, 311, and 338 to help you complete the project. Your responses should be in the project section of your mathematics binder.

Turn to pages 21 to 26 of Assignment Booklet 7B and complete the module project.

MODULE SUMMARY

In this module you have explored the use of vectors in solving problems. You began by recognizing the difference between vectors and scalars. You used vector diagrams to solve simple vector problems; then you focused on solving more involved vector problems in two and three dimensions using computations with vectors.

You applied your knowledge and understanding of vectors to analyse traffic-accident simulations and then applied it to understanding accident reconstruction.

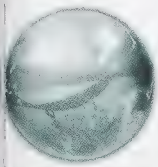
Accident reconstruction is an important part of police work. Police need to be able to determine the distances and directions that vehicles involved in an accident were travelling before and after the accident. Motor-vehicle accident reconstruction involves the knowledge and understanding of vectors. Do you think you might find this type of work interesting?

Diploma Examination Information

Preparing for the Diploma Examination

Congratulations! You have worked hard to get to this point in the course. You have covered a variety of topics and you deserve the opportunity to demonstrate the skills you have developed and the material you have mastered. Are you ready for the diploma examination?

To help you prepare for the diploma examination, go to the Alberta Learning website:



> **http://www.learning.gov.ab.ca/k_12/testing/diploma/dip_default.asp**

This directory posts course expectations, the makeup of the diploma examination, keyed copies of past examinations, preparation guides, calculator policies, and much more. If you have access to the Internet, download copies of these documents or ask your teacher to help you obtain them. The following is a summary of some of the preparation strategies contained in these documents and suggested by the same people who put together the diploma examination you will be writing.

Some of these strategies are also contained in a video titled *Math Factor Review for Senior High Math, Volume 3*. Your school or local library may have a copy of this video. The video includes interviews with students, teachers, and examination personnel.

Here are a few strategies to help prepare for the examination:

- One of the most effective ways of preparing for the diploma examination starts right at the beginning of the course. You should develop your problem-solving and communication skills through daily practice and regular review.

- Find out what will be expected of you on the exam. Work through copies of previous exams. Pay attention to the format and unit weightings. At the time this course was written, the diploma examination consisted of 33 multiple-choice items, 6 machine-scored numerical-response questions, and 3 written-response questions. The multiple-choice and numerical-response questions were worth 65% of the examination; two of the written-response questions were worth 10% each; and the third written-response question was worth 15% of the examination. Check the Alberta Learning website or ask your teacher if this blueprint has changed.

- One of the written-response questions, worth 10%, is based on the student project. Make certain you are familiar with the solution, processes, and procedures contained in the project. Be prepared to address these aspects when you answer this question.

- The questions that appear on the diploma examination have been designed to test the extent to which you know the content and can demonstrate problem-solving skills. Access the website or ask your teacher how you can obtain copies of the curriculum standards. These standards not only outline the content that will be tested, but also describe the depth to which it will be tested. Find out what you will need to know and do to achieve an acceptable standard (50% or higher) and a standard of excellence (80% or higher).

- At least two weeks before the examination, prepare a review schedule. Divide the course into topics and allot time for an adequate review of each topic. It makes sense to spend more time on topics that you may have forgotten, covered earlier in the year, or found difficult. Stick to your schedule as much as possible. There may be times when you cannot adhere to it. Build some flexibility into the schedule to allow for emergencies, and plan to complete your review at least one full day before the test so you can relax and brush up on a few minor points.

- Check and double-check the time and place you will be writing the examination. Also, become familiar with the examination rules and policies. Make sure you have an approved graphing calculator for the examination.

- Find examples of each type of question. Practise these question types. Also, become familiar with the "directing words," such as *compare, describe, evaluate, explain, interpret, justify, prove,* and *solve*. These words are defined for you in information bulletins and tell you how you are expected to answer diploma examination questions that contain these words.

- Make summaries and outlines. Highlight key words and concepts. Develop memory aids to help you recall facts and procedures.

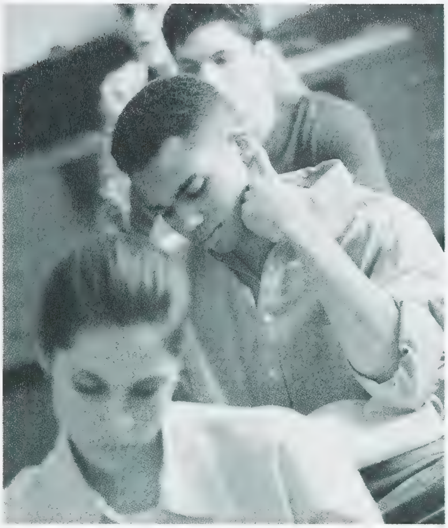
Writing the Applied Mathematics 30 Diploma Examination

Here are a few strategies that will help you do your best on the examination:

- Get a good night's sleep before the examination. You will do better if you are well-rested and relaxed.

- Pack everything you will need the night before. Do not forget your graphing calculator! Remember, all memory locations must be cleared before you enter the examination room. Check your manual or ask your teacher how to clear your calculator's memory. Also, bringing a spare scientific calculator is not a bad idea. You will need soft lead pencils, an eraser, and a clear plastic ruler. Be prepared: bring tissues, cough drops, or candies as required.

- Arrive a few minutes early so you are not rushed and so you enter the exam room in time for all instructions.



- At the beginning of the examination, look over the entire test. Answer questions you are comfortable with first. These questions may jog your memory and help you answer the questions you felt were more difficult.

- Keep track of the time and pace yourself. Remember, you have 3 h to complete the exam, but you should aim for $2\frac{1}{2}$ h so you can use the last $\frac{1}{2}$ h to make any necessary revisions.

- Answer every question. There is no penalty for guessing, and no answer is ever considered foolish. Instead, it is foolish to leave a question unanswered, especially if part marks are awarded!

- If you are stuck on a multiple-choice item, identify the choices you know are incorrect and choose from the remaining choices by using logic and determining what is reasonable.

- In a difficult question, you will find it helpful to highlight key words to help clarify the question and to jog your memory.

- Label diagrams on the examination with the letters and numbers given in the question. Remember, most diagrams are drawn to scale; so, you can use your ruler to identify reasonable choices.

- Don't look for patterns in the multiple-choice section. Have a good reason for changing an answer; often, your first choice is correct.

- When answering a numerical-response question, remember that answers are never negative. If you get a negative answer, try the question again.

- If you have time, prepare an outline for a written-response question and use the outline as a guide in answering the question.

- When completing a written-response question, keep in mind the reader of your response. The marker will want to know how well you understand the problem, how well you use the problem-solving strategies and mathematics involved, and how you communicate your ideas.

- Often, rewriting the statement in a written-response question is a good way to begin. Don't forget to write a summary statement at the end of your solution and to check that you have addressed all aspects of the question.

APPENDIX

Glossary
Suggested Answers
Image Credits

Glossary

bearing: a three-digit angle, measured clockwise from north, between north and the desired direction

displacement: a vector that describes the distance and direction an object moves

equal vectors: vectors that have the same magnitude and direction

equilibrant: a force that acts in the opposite direction to a resultant force to balance a system of forces

heading: the direction toward which a craft is moving

head-to-tail vectors: two vectors drawn so that the tail of the second vector starts at the head of the first vector

magnitude: the size or length of something

opposite vectors: vectors that have the same magnitude but in opposite directions

parallelogram method of adding vectors: a method of finding the sum of two vectors arranged tail-to-tail by completing a parallelogram

resultant: a vector that represents the same direction and magnitude as the sum of two vectors that are combined

scalar: a constant or particular number

triangle method of adding vectors: a method of finding the sum of two vectors arranged head-to-tail by completing a triangle

vector: a line segment that denotes magnitude and direction

zero vector: a vector that has no magnitude

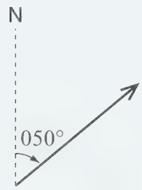
Suggested Answers

Activity 1: Vectors and Scalars

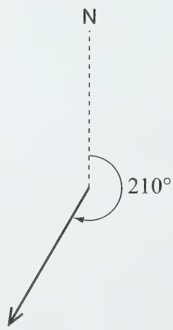
1. A geometric vector is a directed line segment that shows length and direction.

2. The person has used a scalar quantity.

3. a.



b.



4. **Textbook exercises 1 and 2 of "Exercises: Checking Your Skills," p. 307**

   1. The statements that can be described by vectors are a., d., g., and h.

   2. The quantities that are scalars are a., b., d., e., f., and g.

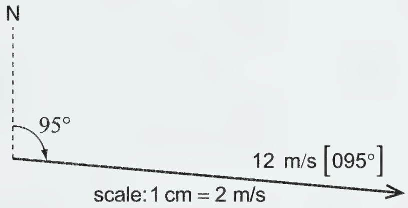
5. **Textbook exercises 3.a., 3.b., 3.c., 4.a., 4.c., 4.d., 5.b., 5.c., and 5.d. of "Exercises: Checking Your Skills," pp. 308 and 309**

   3. **a.** $2.3 \times 10 = 23$ m/s east **b.** $3.2 \times 5 = 16$ m west **c.** $2.7 \times 100 = 270$ km/h south

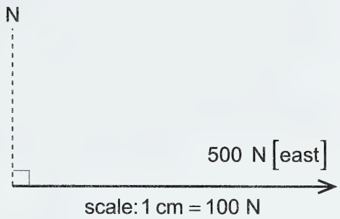
   4. **a.** $2.0 \times 20 = 40$ m/s north **c.** $3.3 \times 10 = 33$ km/h southeast

   d. $49 \times 1 = 49$ m/s^2 northeast

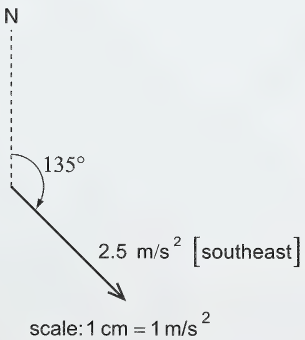
   5. **b.** Let 1 cm = 2 m/s. The segment will then be $12 \div 2 = 6$ cm in length. Using a protractor, mark a direction that is 95° east of north. Construct a line segment that is 6 cm in length, adding an arrowhead pointing in the desired direction. Finally, label your diagram.



   c.

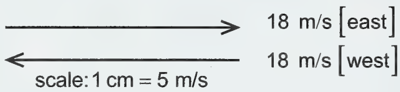


   d.



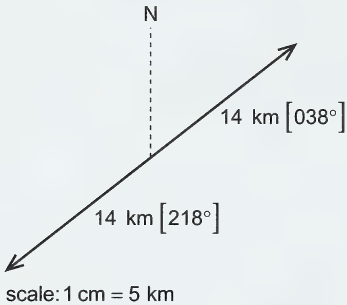
6. **Textbook exercises 6, 7, and 8 of "Exercises: Checking your Skills," p. 309**

   6. **a.** The opposite vector to 18 m/s [east] is 18 m/s [west].

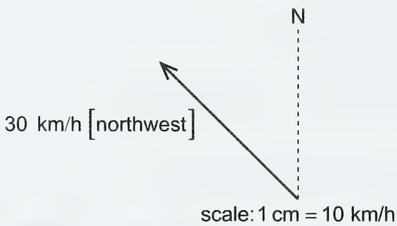


   b. $180° + 38° = 218°$

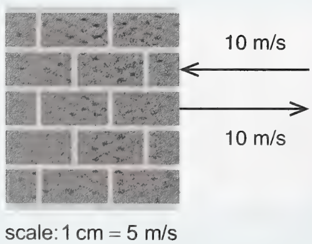
   The opposite vector to 14 km [038°] is 14 km [218°].



   7. The assumption here is that you construct the vector from the initial position of the hurricane to the coast of Florida.



   8.



7. **Textbook exercise “Communicating the Ideas,” p. 309**

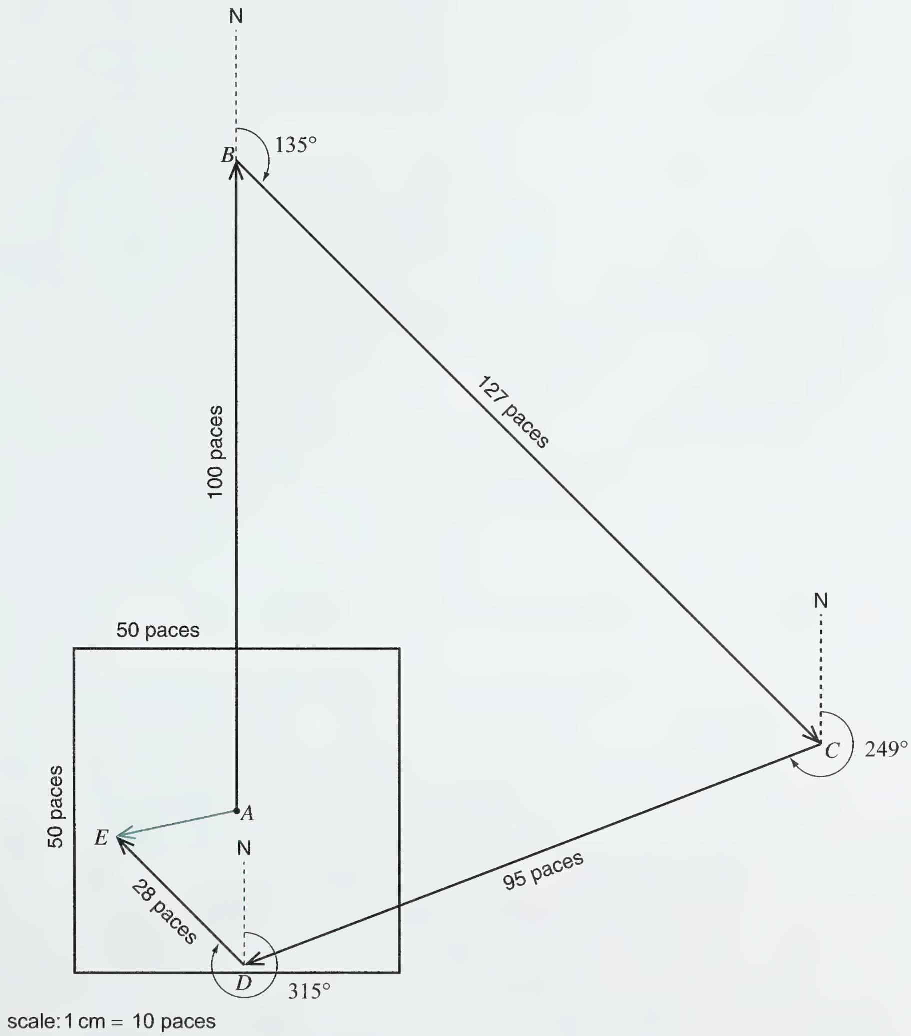
Answers will vary. A sample note is given.

A scalar indicates only magnitude, while a vector indicates magnitude and direction. An example of a scalar is 30 km/h. An example of the same quantity as a vector is 30 km/h [southeast]. The scalar quantity could represent, for example, a wind speed of 30 km/h, whereas the vector quantity would represent a wind speed of 30 km/h heading southeast.

Activity 2: Adding Vectors Using Scale Diagrams

1. **Textbook exercises 1, 2, and 3 of “Investigation 1: Adding Vectors That Represent Changes in Position,” p. 312**

1. to 3. Your diagram should look like the following.



Activity 2 (continued)

$$\overrightarrow{AE} = 1.9 \text{ cm} \times 10 \text{ paces/cm}$$
$$= 19 \text{ paces}$$

The angle of $\overrightarrow{AE}$ is $360° - 102° = 258°$. Therefore, the displacement of $\overrightarrow{AE}$ is 19 paces [258°].

2. Textbook exercises 1, 2, and 3 of "Exercises: Checking Your Skills," p. 318

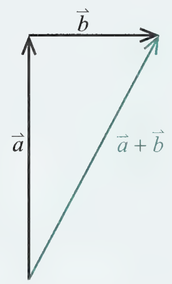
Note: Answers may vary slightly due to slight measurement differences.

1. a. Total distance $= \left(15 \text{ km/h} \times \frac{30}{60} \text{ h}\right) + \left(12 \text{ km/h} \times \frac{20}{60} \text{ h}\right)$

$$= 7.5 \text{ km} + 4 \text{ km}$$
$$= 11.5 \text{ km}$$

Jack has jogged 11.5 km in total.

b.

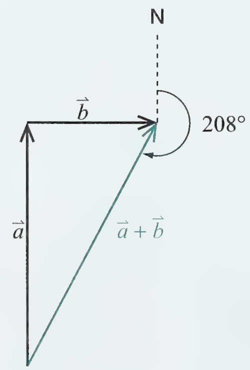


scale: 1 cm = 2 km

$$\vec{a} + \vec{b} = 4.2 \text{ cm} \times 2 \text{ km/cm}$$
$$= 8.4 \text{ km}$$

Note: The answer in the textbook is 8.5 km. Because most rulers measure to the nearest millimetre, 8.4 km and 8.6 km are also acceptable answers.

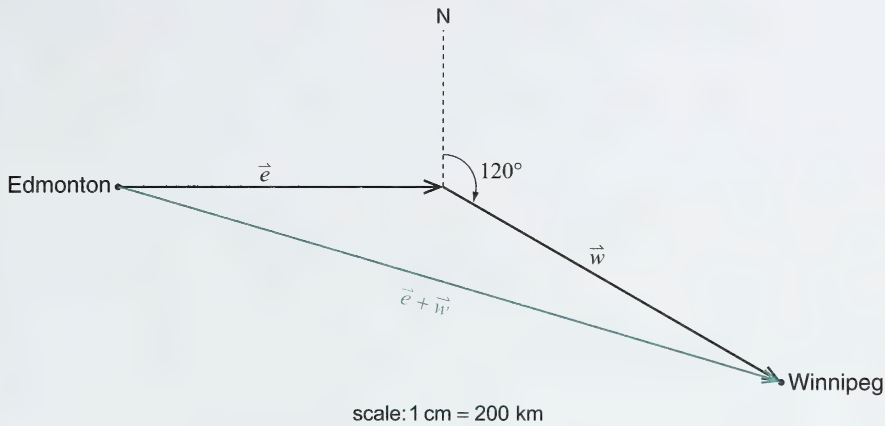
c.



scale: 1 cm = 2 km

Jack needs to travel at a bearing of [208°] in order to return to the start by the shortest path.

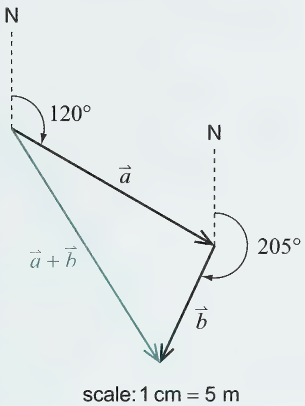
2.



$$\begin{aligned}\vec{e}+\vec{w} &= 10.7 \text{ cm} \times 200 \text{ km/cm} \\ &= 2140 \text{ km}\end{aligned}$$

The displacement of the jet from Edmonton is 2140 km [107°].

3.

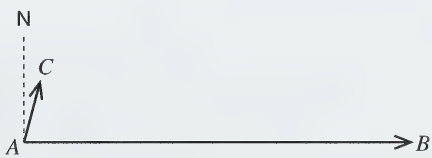


$$\begin{aligned}\vec{a}+\vec{b} &= 4.3 \text{ cm} \times 5 \text{ m/cm} \\ &= 21.5 \text{ m}\end{aligned}$$

The diver is 21.5 m [148°] from the starting point.

3. Textbook exercises 1 and 2 of "Investigation 2: The Parallelogram Method for Adding Two Vectors," p. 313

1. Your diagram should be similar to the following.



Activity 2 (continued)

2. Your diagram should be similar to the following.

4. Textbook exercises 1, 2, and 3 of "Discussing the Ideas," p. 317

1. Two different vectors are equal if they are equal in length, are parallel, and have the same direction.

2. An advantage of using scale diagrams to add vectors is that you have a visual representation of the vectors and that trigonometric calculations are not required. A disadvantage is that the values determined may not be very accurate.

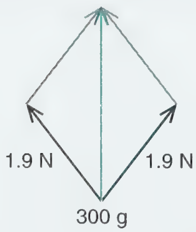
3. When adding three or more vectors using the triangle method, begin by drawing the first vector at the given direction; then draw each subsequent vector at the given direction with the tail of that vector starting at the head of the preceding vector. Determine the resultant by drawing the vector from the tail of the first vector to the head of the last vector.

   When using the parallelogram method, begin by drawing the first two vectors; then draw the resultant vector. Then draw the next vector starting at the head of the resultant, and draw a new resultant vector. Continue by adding vectors and obtaining new resultants until all the vectors have been added and the final resultant is determined.

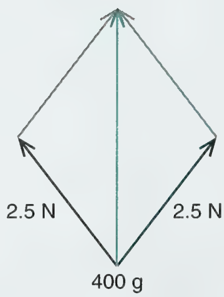
5. Textbook exercises 6, 7, and 8 of "Investigation 3: A Study of Forces," p. 315

6. Use scale diagrams to determine the resultant in each case. Your answers may vary, but they should be similar to the following.

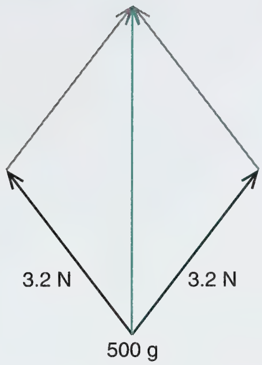
   $\theta = 75° \left(\textbf{scale : 1 cm = 1 N}\right)$



$$\text{Resultant force} = 3.0 \text{ cm} \times 1 \text{ N/cm}$$
$$= 3.0 \text{ N}$$

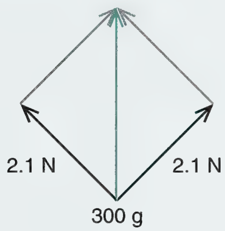


$$\text{Resultant force} = 4.0 \text{ cm} \times 1 \text{ N/cm}$$
$$= 4.0 \text{ N}$$

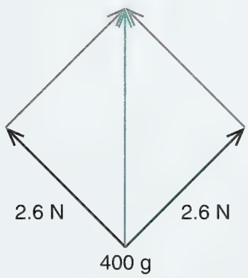


$$\text{Resultant force} = 5.1\ \text{cm} \times 1\ \text{N/cm}$$
$$= 5.1\ \text{N}$$

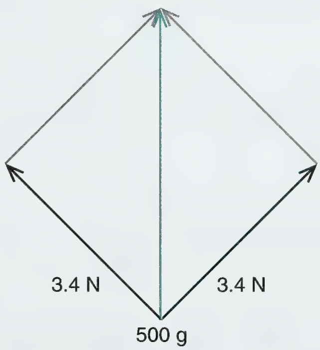
$\theta = 90°\,(\text{scale} : 1\ \text{cm} = 1\ \text{N})$



$$\text{Resultant force} = 3.0\ \text{cm} \times 1\ \text{N/cm}$$
$$= 3.0\ \text{N}$$

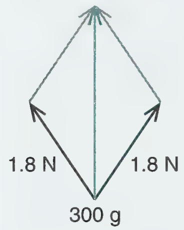


$$\text{Resultant force} = 3.7\ \text{cm} \times 1\ \text{N/cm}$$
$$= 3.7\ \text{N}$$

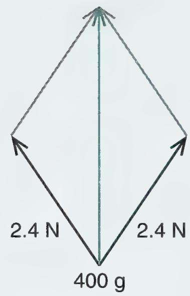


$$\text{Resultant force} = 4.8\ \text{cm} \times 1\ \text{N/cm}$$
$$= 4.8\ \text{N}$$

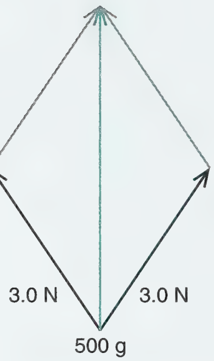
$\theta = 68° \left(\text{scale} : 1 \text{ cm} = 1 \text{ N}\right)$



$$\begin{aligned}\text{Resultant force} &= 3.0 \text{ cm} \times 1 \text{ N/cm} \\ &= 3.0 \text{ N}\end{aligned}$$



$$\begin{aligned}\text{Resultant force} &= 4.0 \text{ cm} \times 1 \text{ N/cm} \\ &= 4.0 \text{ N}\end{aligned}$$



$$\begin{aligned}\text{Resultant force} &= 5.0 \text{ cm} \times 1 \text{ N/cm} \\ &= 5.0 \text{ N}\end{aligned}$$

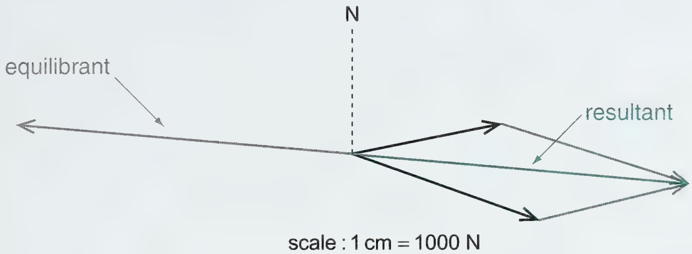
The following table summarizes the resultant force for each mass at the given angles. The forces are the same or close to the values given for the force for each mass in the answer to exercise 1. (Refer to page 411 of the textbook.)

Mass (g)	Resultant Force (N)		
	[illegible]	75°	[illegible]
300	3.0	3.0	3.0
400	4.0	4.0	3.7
500	5.0	5.0	4.8

7. You should use a wire with more slack to hang a heavy picture. When you have more slack in the wire, the force on each side of the wire is less than when the wire is taunt.

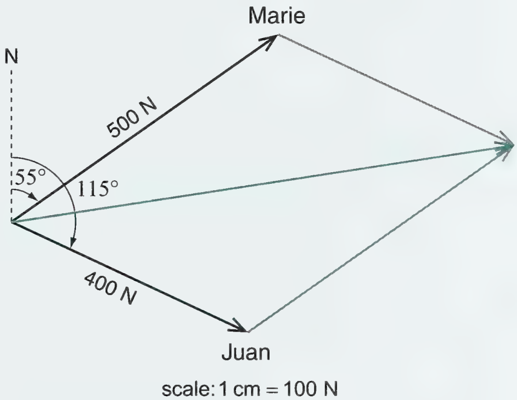
8. For a given mass, as the angle between the strings holding the two scales increases, the force on each string increases. Also, the amount of force on each string increases as the mass increases.

6.



7. a. Textbook exercises 6, 7, and 8 of "Exercises: Checking Your Skills," pp. 318 and 319

6. a.



An estimate of the magnitude and direction of the resultant is 800 N at a bearing of about [080°].

b. $\text{Resultant force} = 7.8 \text{ cm} \times 100 \text{ N/cm}$

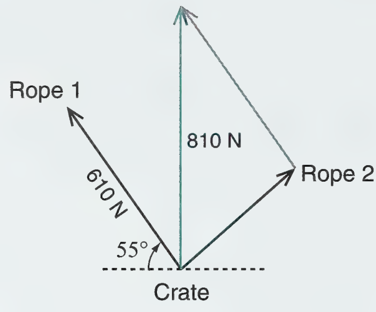
$= 780 \text{ N}$

The magnitude and direction of the resultant force is 780 N [081°].

Activity 2 (continued)

7. **Diagram 1**

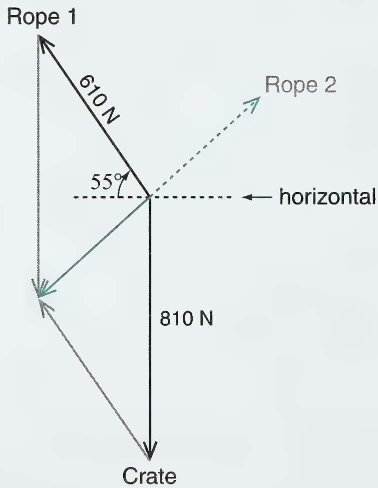
Since 810 N of force is required to suspend the crate, the resultant force of the two ropes must be 810 N up.



scale: 1 cm = 200 N

Diagram 2

Since the other rope must act to hold the crate from moving, it must be the equilibrant of the resultant and act in an opposite direction to the resultant.

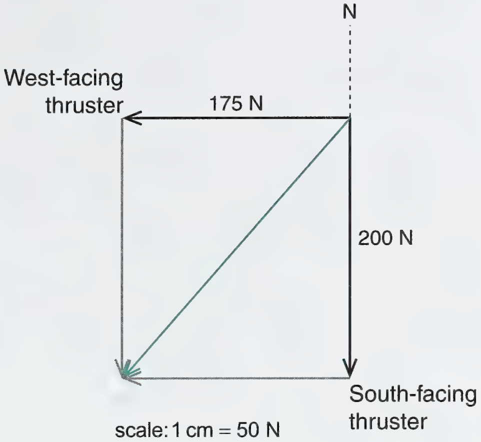


scale: 1 cm = 200 N

$$\begin{aligned}\text{Rope } 2 &= 2.3 \text{ cm} \times 200 \text{ N/cm} \\ &= 460 \text{ N}\end{aligned}$$

The force exerted by the other rope is 460 N. The rope would make an angle of 41° with the horizontal.

8.

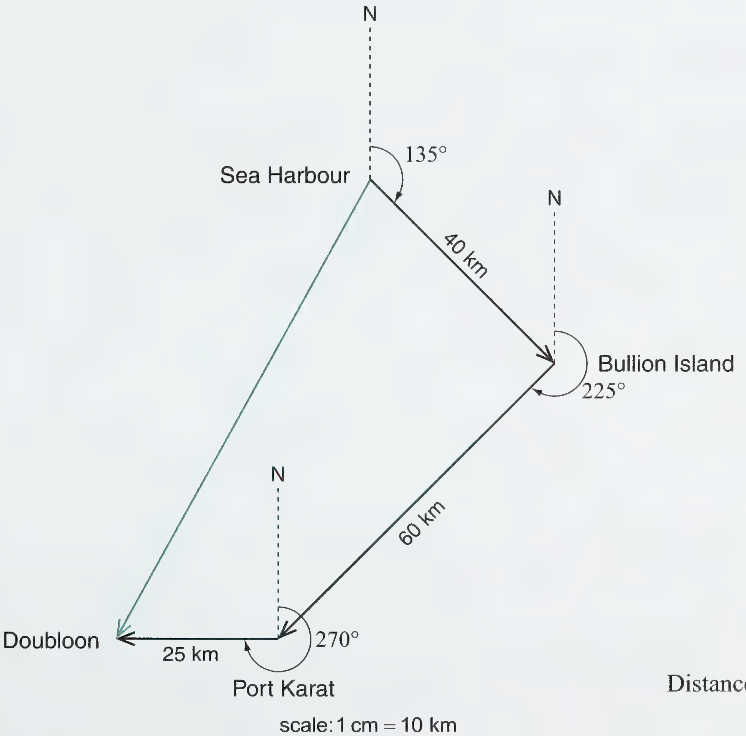


$$\text{Resultant} = 5.3 \text{ cm} \times 50 \text{ N/cm}$$
$$= 265 \text{ N}$$

The resultant thrust on the spaceship is 265 N [221°].

b. Textbook exercise 10 of "Exercises: Extending Your Thinking," p. 319

10.



$$\text{Distance} = 8.1 \text{ cm} \times 10 \text{ km/cm}$$
$$= 81 \text{ km}$$

The distance and bearing of Doubloon from Sea Harbour is 81 km [209°].

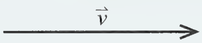
Activity 2 (continued)

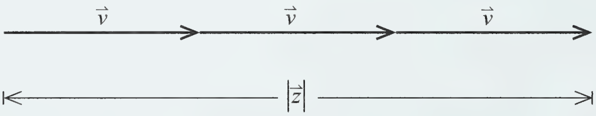
8. Textbook exercise "Communicating the Ideas," p. 319

The triangle method and the parallelogram method are similar in that when the vectors in the parallelogram diagram are moved to the proper position, they can be added using the triangle method. The methods are different in that the triangle method is used when vectors are placed head-to-tail, whereas the parallelogram method is used when vectors are placed tail-to-tail.

Activity 3: Multiplying a Vector by a Scalar

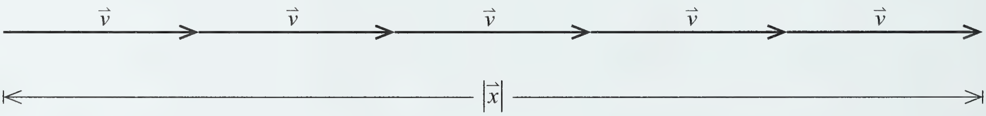
1. Textbook exercises 1 to 9 of "Investigation 1: Scalar Multiples of a Vector," p. 320

1. 

2. 

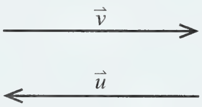
3. The direction of $\vec{z}$ is [090°], and the magnitude is 9 cm.

   $\therefore\ \vec{z} = 9\ \text{cm}\,[090°]$

4. 

5. The direction of $\vec{x}$ is [090°], and the magnitude is 15 cm.

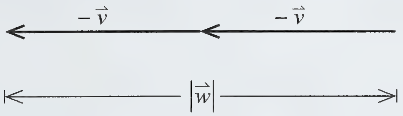
   $\therefore\ \vec{x} = 15\ \text{cm}\,[090°]$

6. 

7. The direction of $\vec{u}$ is [270°], and the magnitude is 3 cm.

   $\therefore\ \vec{u} = 3\ \text{cm}\,[270°]$

8.



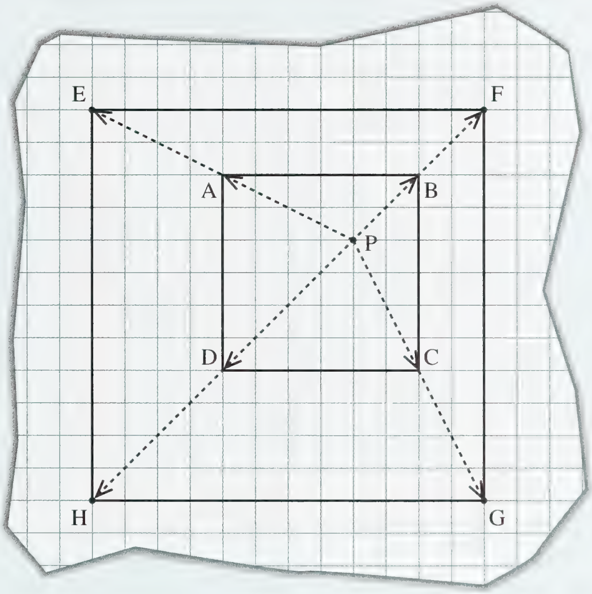
9. The direction of $\vec{w}$ is [270°], and the magnitude is 6 cm.

$\therefore\ \vec{w} = 6\ \text{cm}\left[270°\right]$

2. Although the statements in the coloured box are written in a mathematical language, they should say the same thing about vector multiplication by a scalar as your summary. The additional statement states that when a vector is multiplied by zero, the result is a vector that has no magnitude, called the zero vector.

3. **Textbook exercises 1 to 6 of "Investigation 2, Enlarging Figures," p. 321**

1. to 3.



4. Figure EFGH is a square with sides measuring 6 cm. Square EFGH encompasses square ABCD.

Activity 3 (continued)

5.

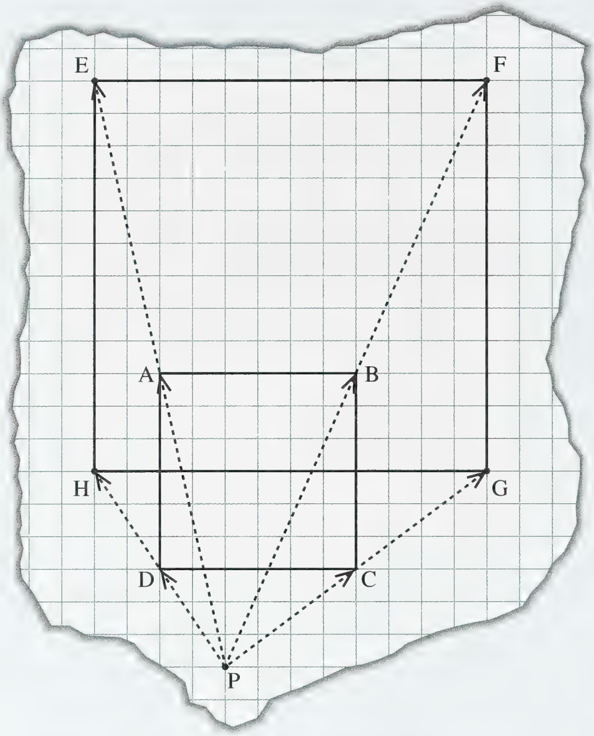


Figure EFGH is a square with sides measuring 6 cm. In this case, only part of square ABCD is covered by square EFGH. Depending on the placement of P, square EFGH may cover more of square ABCD or may cover none of square ABCD.

6.

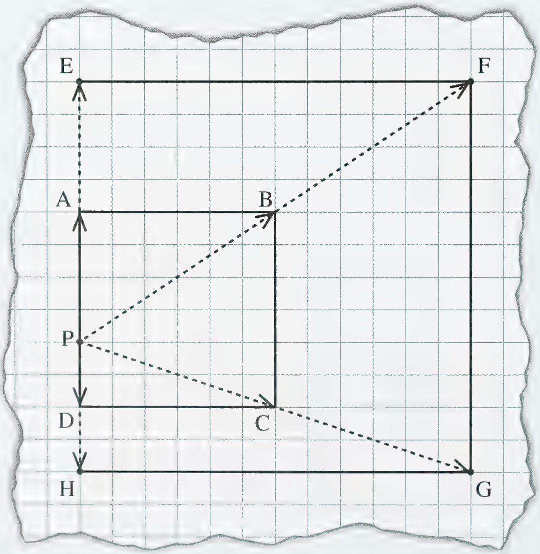
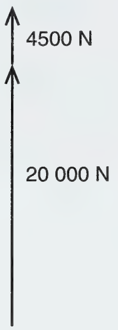


Figure EFGH is a square with sides measuring 6 cm. One side of square EFGH is on the corresponding side of square ABCD. The other sides of square EFGH appear outside of square ABCD, encompassing the square.

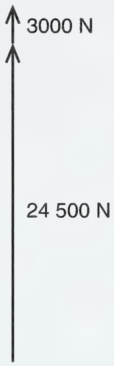
4. a. **Textbook exercises 2, 3, and 4 of "Exercises: Checking Your Skills," pp. 322 and 323**

2. The scale of each diagram is 1 mm = 500 N or 1 cm = 5000 N.

a. $6 \times 750\ \text{N} = 4500\ \text{N}$

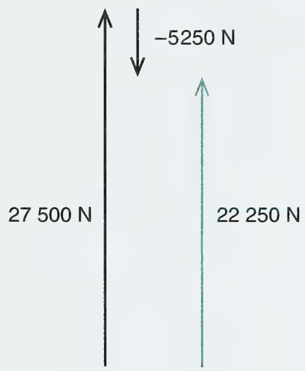


b. $4 \times 750\ \text{N} = 3000\ \text{N}$



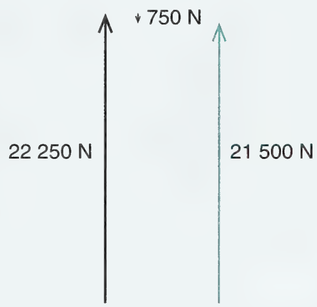
c. A net of 7 people leave the elevator.

$-7 \times 750\ \text{N} = -5250\ \text{N}$



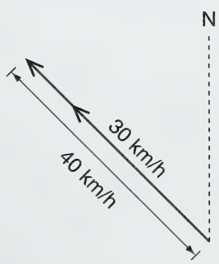
d. A net of 1 person leaves the elevator.

$-1 \times 750\ \text{N} = -750\ \text{N}$



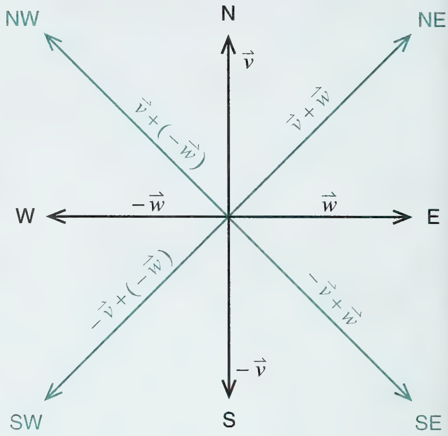
(The two vectors to be added and the resultant are drawn separately to assist in seeing how the resultant is found.)

3. scale: 1 cm = 10 km/h



Activity 3 (continued)

4. Draw a diagram showing the unit vectors $\vec{v}$ and $\vec{w}$. Extend it to show the unit vectors $-\vec{v}$ and $-\vec{w}$. Exercises 4.a. to 4.d. ask about vectors pointing northeast, southeast, southwest, and northwest. A diagram like the one on the right shows how the unit vectors can be combined to get vectors with the required directions.



a. For $\vec{u}$ to point northeast, the diagram suggests $\vec{u}=(1)\vec{v}+(1)\vec{w}$ as one possibility. Since multiplying a vector by a positive scalar does not change its direction, any positive multiple of $(1)\vec{v}+(1)\vec{w}$ will have a direction of northeast. Let p be any positive number; therefore, $p\vec{v}+p\vec{w}$ would be a suitable vector. Since you are to answer in terms of $k\vec{v}+m\vec{w}$, $k=p$ and $m=p$, where $p>0$. Therefore, $k=m$, where $k>0$ $(\text{or } m>0)$ will result in $\vec{u}$ pointing northeast.

b. For $\vec{u}$ to point northwest, the diagram suggests $\vec{u}=(1)\vec{v}+(-1)\vec{w}$ as one possibility. Since multiplying a vector by a positive scalar does not change its direction, any positive multiple of $(1)\vec{v}+(-1)\vec{w}$ will have a direction of northwest. Let p be any positive number; therefore, $p\vec{v}+(-p)\vec{w}$ would be a suitable vector. Since you are to answer in terms of $k\vec{v}+m\vec{w}$, $k=p$ and $m=-p$ $(\text{and } -m=p)$, where $p>0$. Therefore, $k=-m$, where $k>0$ $(\text{and } m<0)$ will result in $\vec{u}$ pointing northwest.

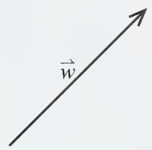
c. For $\vec{u}$ to point southeast, the diagram suggests $\vec{u}=(-1)\vec{v}+(1)\vec{w}$ as one possibility. Since multiplying a vector by a positive scalar does not change its direction, any positive multiple of $(-1)\vec{v}+(1)\vec{w}$ will have a direction of southeast. Let p be any positive number; therefore, $(-p)\vec{v}+p\vec{w}$ would be a suitable vector. Since you are to answer in terms of $k\vec{v}+m\vec{w}$, $k=-p$ $(\text{and } -k=p)$, where $p>0$. Therefore, $-k=m$, where $m>0$ $(\text{and } k<0)$ will result in $\vec{u}$ pointing southeast.

d. For $\vec{u}$ to point southwest, the diagram suggests $\vec{u} = (-1)\vec{v} + (-1)\vec{w}$ as one possibility. Since multiplying a vector by a positive scalar does not change its direction, any positive multiple of $(-1)\vec{v} + (-1)\vec{w}$ will have a direction of southwest. Let p be any positive number; therefore, $(-p)\vec{v} + (-p)\vec{w}$ would be a suitable vector. Since you are to answer in terms of $k\vec{v} + m\vec{w}$, $k = -p$ and $m = -p$, where $p > 0$. Therefore, $k = m$, where $k < 0$ and $m < 0$ will result in $\vec{u}$ pointing southwest.

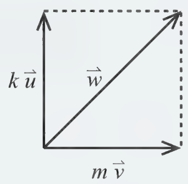
b. Textbook exercises 5.a. and 5.c. of "Exercises: Extending Your Thinking," p. 323

5. a. Method 1: Using a Scale Diagram

Step 1: Draw $\vec{w}$ 3 cm in length and pointing northeast.



Step 2: Complete a rectangle and measure the length of $k\vec{u}$ and $m\vec{v}$.



$k\vec{u} = 2.1$ cm

$m\vec{v} = 2.1$ cm

Step 3: Determine k and m.

$$k\vec{u} = 2.1 \text{ cm} \qquad m\vec{v} = 2.1 \text{ cm}$$

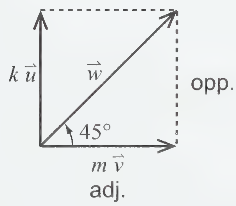
$$k(1 \text{ cm}) = 2.1 \text{ cm} \qquad m(1 \text{ cm}) = 2.1 \text{ cm}$$

$$k = 2.1 \qquad m = 2.1$$

Method 2: Using Trigonometry

$$\sin 45° = \frac{\text{opp.}}{\text{hyp.}} \qquad \cos 45° = \frac{\text{adj.}}{\text{hyp.}}$$

$$\sin 45° = \frac{k\vec{u}}{\vec{w}} \qquad \cos 45° = \frac{m\vec{v}}{\vec{w}}$$



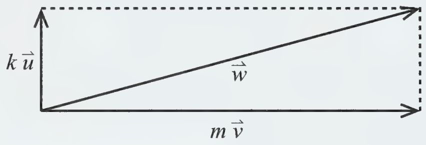
$$k\vec{u} = \vec{w}\sin 45° \qquad m\vec{v} = \vec{w}\cos 45°$$

$$k(1) = 3\sin 45° \qquad m(1) = 3\cos 45°$$

$$k \doteq 2.1 \qquad m \doteq 2.1$$

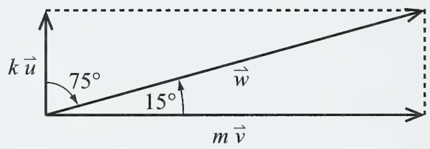
c. **Method 1: Using a Scale Diagram**

Draw $\vec{w} = 6 \text{ cm} [075°]$, complete a rectangle, and measure the lengths of $k\vec{u}$ and $m\vec{v}$.



$$\begin{aligned} k\vec{u} &= 1.6 \text{ cm} \\ k(1 \text{ cm}) &= 1.6 \text{ cm} \\ k &= 1.6 \end{aligned} \qquad \begin{aligned} m\vec{v} &= 5.8 \text{ cm} \\ m(1 \text{ cm}) &= 5.8 \text{ cm} \\ m &= 5.8 \end{aligned}$$

Method 2: Using Trigonometry



$$\cos 75° = \frac{\text{adj.}}{\text{hyp.}}$$

$$\cos 75° = \frac{k\vec{u}}{\vec{w}}$$

$$\begin{aligned} k\vec{u} &= \vec{w}\cos 75° \\ k(1) &= 6\cos 75° \\ k &\doteq 1.6 \end{aligned}$$

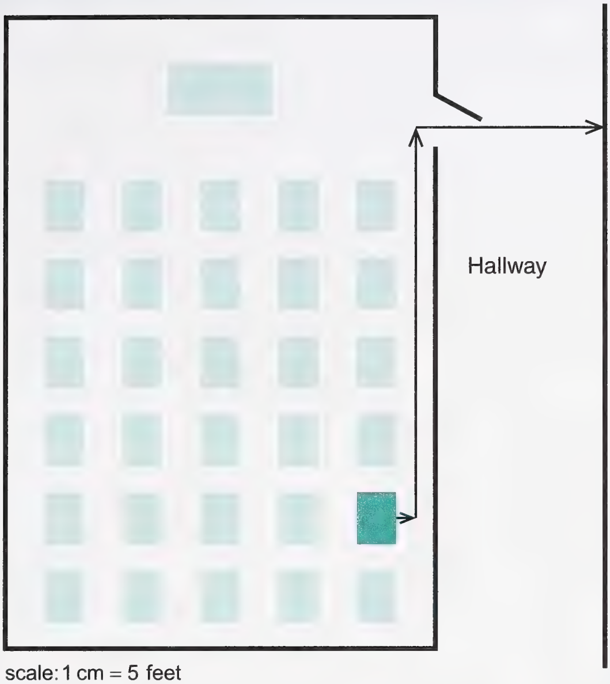
$$\cos 15° = \frac{\text{adj.}}{\text{hyp.}}$$

$$\cos 15° = \frac{m\vec{v}}{\vec{w}}$$

$$\begin{aligned} m\vec{v} &= \vec{w}\cos 15° \\ m(1) &= 6\cos 15° \\ m &\doteq 5.8 \end{aligned}$$

5. Vectors are used to indicate magnitude and direction. Magnitude and direction are important in programming robots to do certain tasks.

6. a.



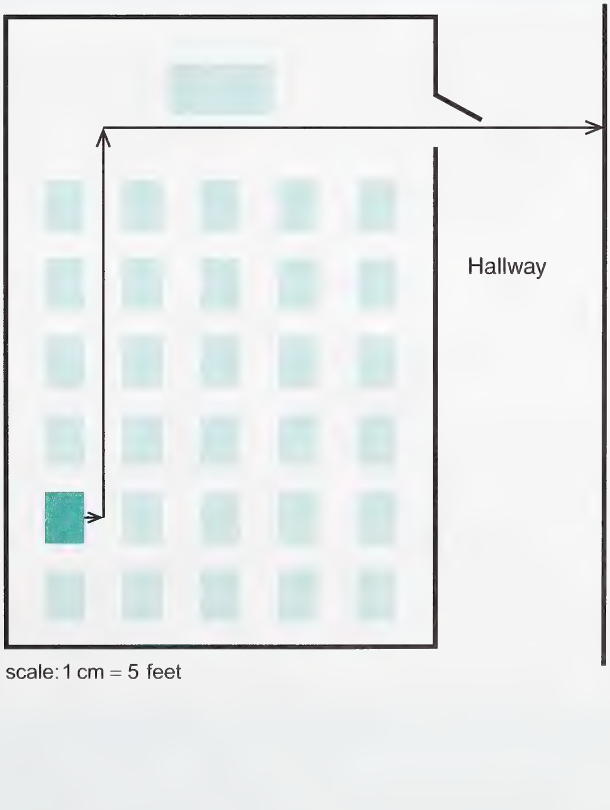
scale: 1 cm = 5 feet

b. It takes 3 straight-line paths.

c. $$\begin{aligned}\text{Distance} &= (0.3\ \text{cm} + 6.0\ \text{cm} + 2.9\ \text{cm}) \\ &\quad \times 5\ \text{feet/cm} \\ &= 9.2\ \text{cm} \times 5\ \text{feet/cm} \\ &= 46\ \text{feet}\end{aligned}$$

The length of the path is 46 feet.

7. The pathway would be in the same direction for all three straight-line paths as in exercise 6. This is shown in the following diagram.



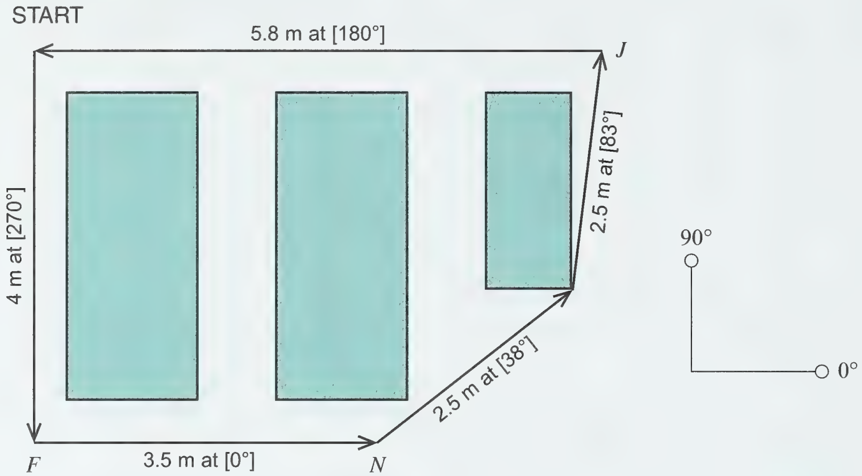
scale: 1 cm = 5 feet

8. You could use right, left, front, rear, north, south, east, west; or you could use bearing.

9. The purpose of establishing a convention is so everyone can describe direction in the same way.

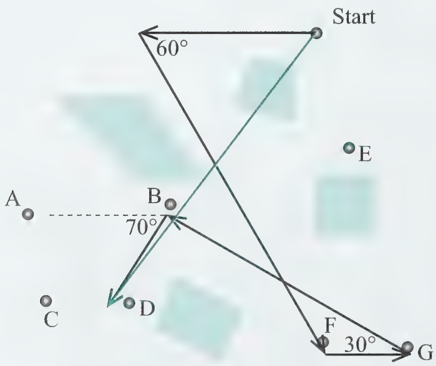
10. Project Book exercises 4 and 5 of "Getting Started," pp. 132 and 133

4. Your completed template should be similar to the following. (This diagram is not drawn to the same scale as the one in the Project Book.)



Five vectors are needed. You could have used more vectors if you went between the three shaded areas. The minimum total route is 18.3 m. If the robot followed the route backward, the only difference is that the direction is opposite for each vector. There are an infinite number of ways to visit the points if you do not want to travel the minimum distance.

5. Your completed template should be similar to the following.



The robot ends up between points C and D. The resultant vector is 5.3 m at 233°. The resultant vector cannot be used as a single vector to go from Start to the terminal point because there is an object in the way. Situations where it is important to know the shortest path between two points are delivery jobs and networking computers. Yes, you can change the order of some of the vectors and end up at the same point. You can use two vectors to end up at point C by going from Start to almost the left edge of the diagram then down to point C. You can reach point E from Start with one vector. Yes, all the other points can be reached in two vectors.

11. Textbook exercise "Communicating the Ideas," p. 323

Multiplying a vector by a number is similar to multiplying real numbers. Both are done by repeated addition. For example, multiplying a given vector by the scalar 3 is the same as adding the vector 3 times. When the scalar is positive, the original direction of the vector is maintained. When the scalar is negative, the direction of the new vector is opposite the original vector. Multiplying real numbers differs in that real numbers do not have direction.

Activity 4: Solving Vector Problems by Computation

1. Textbook exercises 1, 2, and 4 of "Practise Your Prior Skills," p. 324

1.

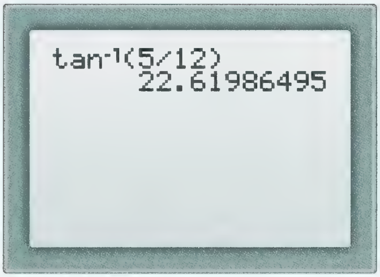
$$\tan A = \frac{BC}{AB}$$

$$= \frac{5}{12} \text{ or about } 0.4167$$

$$\angle A = \tan^{-1}(\tan A)$$

$$= \tan^{-1}\left(\frac{5}{12}\right)$$

$$\doteq 23°$$



$$(AC)^2 = (AB)^2 + (BC)^2$$

$$(AC)^2 = 12^2 + 5^2$$

$$(AC)^2 = 144 + 25$$

$$(AC)^2 = 169$$

$$AC = \sqrt{169}$$

$$= 13 \text{ cm}$$

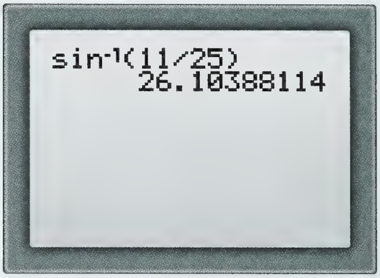
2. $\sin R = \dfrac{PQ}{PR}$

$= \dfrac{11}{25}$ or 0.44

$\angle R = \sin^{-1}(\sin R)$

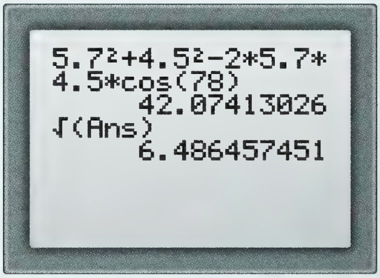
$= \sin^{-1}\left(\dfrac{11}{25}\right)$

$\doteq 26°$



$$(PR)^2 = (QR)^2 + (PQ)^2$$
$$(QR)^2 = (PR)^2 - (PQ)^2$$
$$(QR)^2 = 25^2 - 11^2$$
$$(QR)^2 = 625 - 121$$
$$(QR)^2 = 504$$
$$QR = \sqrt{504}$$
$$\doteq 22.4 \text{ cm}$$

4. $$b^2 = a^2 + c^2 - 2ac\cos B$$
$$(AC)^2 = (BC)^2 + (AB)^2 - 2(BC)(AB)\cos 78°$$
$$(AC)^2 = 5.7^2 + 4.5^2 - 2(5.7)(4.5)\cos 78°$$
$$(AC)^2 \doteq 42.074\,130\,26$$
$$AC \doteq 6.486\,457\,451$$



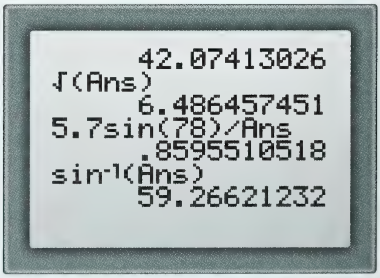
The length of AC is about 6.5 cm.

Use the Sine Law to determine $\angle A$.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{5.7}{\sin A} \doteq \frac{6.486\,457\,451}{\sin 78°}$$

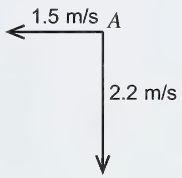
← Use the unrounded value for a more accurate answer.

$$\sin A \doteq \frac{5.7\sin 78°}{6.486\,457\,451}$$
$$\sin A \doteq 0.859\,551\,051\,8$$
$$\angle A \doteq 59°$$

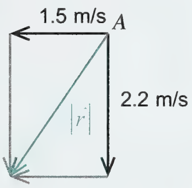


2. a. Textbook exercises 1, 2, and 6 of "Exercises: Checking Your Skills," pp. 328 and 329

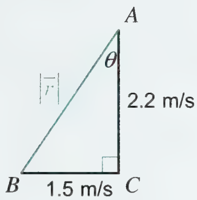
1. Step 1: Sketch the situation and all of the vectors involved.



Step 2: Sketch the corresponding vector diagram, including the resulant, $\vec{r}$.



Step 3: Extract a simple triangle from the vector sketch, showing which sides and angles are to be calculated.



Side c and $\angle A$ need to be determined.

Activity 4 (continued)

Step 4: Solve for the desired unknowns.

Use the Pythagorean Theorem to find c.

$$|\vec{r}|^2 = a^2 + b^2$$

$$|\vec{r}|^2 = (1.5)^2 + (2.2)^2$$

$$|\vec{r}|^2 = 2.25 + 4.84$$

$$|\vec{r}|^2 = 7.09$$

$$|\vec{r}| \doteq 2.7 \text{ m/s}$$

Determine $\angle A$.

$$\tan A = \frac{a}{b}$$

$$\tan A = \frac{1.5}{2.2}$$

$$\angle A = \tan^{-1}\left(\frac{1.5}{2.2}\right)$$

$$\doteq 34°$$

Step 5: Interpret your solution.

$$\text{Bearing} \doteq 180° + 34°$$

$$\doteq 214°$$

Your resultant velocity is approximately 2.7 m/s [214°].

2. $|\vec{r}|^2 = a^2 + c^2$

$$|\vec{r}|^2 = 45^2 + 250^2$$

$$|\vec{r}|^2 = 2025 + 62\,500$$

$$|\vec{r}|^2 = 64\,525$$

$$|\vec{r}| \doteq 254.0 \text{ km/h}^2$$

$$\tan A = \frac{a}{c}$$

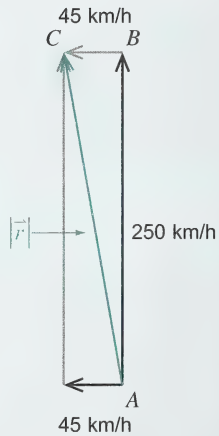
$$\tan A = \frac{45}{250}$$

$$\angle A = \tan^{-1}\left(\frac{45}{250}\right)$$

$$\doteq 10°$$

$$\therefore \text{Bearing} \doteq 360° - 10°$$

$$\doteq 350°$$



The resultant velocity of the aircraft is about 254.0 km/h [350°].

6. $|\vec{r}|^2 = a^2 + c^2$

$|\vec{r}|^2 = 510^2 + 755^2$

$|\vec{r}|^2 = 260\ 100 + 570\ 025$

$|\vec{r}|^2 = 830\ 125$

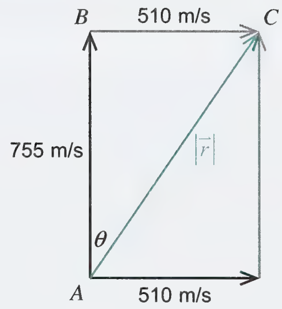
$|\vec{r}| \doteq 911.1$ m/s

$\tan A = \frac{a}{c}$

$\tan A = \frac{510}{755}$

$\angle A = \tan^{-1}\left(\frac{510}{755}\right)$

$\doteq 34°$



The resultant velocity of the rocket is about 911.1 m/s [34°].

b. Textbook exercise 11 of "Exercises: Extending Your Thinking," p. 329

11. The two vectors to be drawn are the wind vector and the straight-line vector from Dauphin to Brandon. Because the vectors need to be in the same units, convert the distance vector (200 km [south]) to a speed vector needed to travel 200 km in 0.5 h. Therefore, the straight-line vector from Dauphin to Brandon is 400 km/h [south].

Now, determine the resultant speed and heading of the aircraft needed to arrive in Brandon in the allotted time.

$|\vec{r}|^2 = a^2 + b^2$

$|\vec{r}|^2 = 45^2 + 400^2$

$|\vec{r}|^2 = 2025 + 160\ 000$

$|\vec{r}|^2 = 162\ 025$

$|\vec{r}| \doteq 402.5$ km/h

$\tan A = \frac{a}{b}$

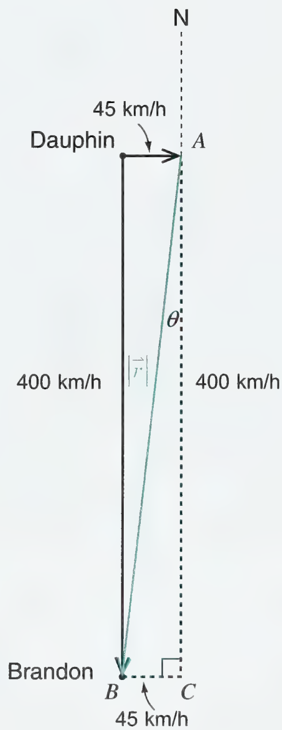
$\tan A = \frac{45}{400}$

$\angle A = \tan^{-1}\left(\frac{45}{400}\right)$

$\doteq 6°$

$\therefore$ Bearing $\doteq 180° + 6°$

$\doteq 186°$



The plane must travel approximately 402.5 km/h [186°].

3. The Cosine Law is used to calculate the resultant in Example 2 because two sides and the included angle are the known parts in the triangle.

4. $\angle BAD$ is the sum of two angles, one of which is given as a bearing (85°). The other is determined by subtracting the given bearing of 330° from the total distance around the north line (360°).

5. $180° - 115° = 65°$

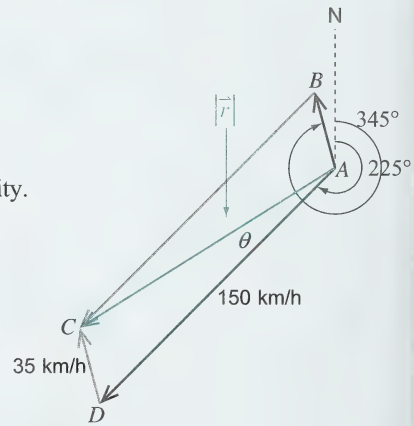
   $\angle ADC$ is equal to 65° since the sum of two adjacent angles in a parallelogram are equal to 180°.

6. The heading of the resultant vector has been determined to be 15.2°. This is the amount that the wind has shifted the direction of the aircraft from its intended heading. The new heading is the sum of the intended heading (330°) and the amount of the shift.

7. **Textbook exercises 4 and 8 of "Exercises: Checking Your Skills," pp. 328 and 329**

   4. $$\begin{aligned}\angle BAD &= 345° - 225°\\ &= 120°\end{aligned}$$

   $$\begin{aligned}\therefore\ \angle ADC &= 180° - 120°\\ &= 60°\end{aligned}$$ ← Two adjacent angles in a parallelogram are equal to 180°.



   Use the Cosine Law to determine the aircraft's resultant velocity.

   $$|\vec{r}|^2 = (CD)^2 + (AD)^2 - 2(CD)(AD)\cos\angle ADC$$

   $$|\vec{r}|^2 = 35^2 + 150^2 - 2(35)(150)\cos 60°$$

   $$|\vec{r}|^2 = 18\ 475$$

   $$\begin{aligned}|\vec{r}| &\doteq 135.922\ 772\ 22\\ &\doteq 135.9\ \text{km/h}\end{aligned}$$

Use the Sine Law to determine $\angle CAD$.

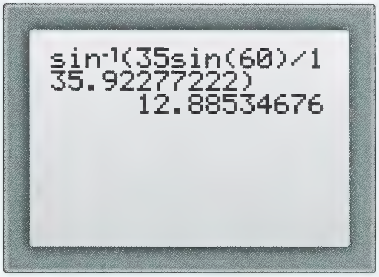
$$\frac{CD}{\sin \angle CAD} = \frac{AC}{\sin \angle ADC}$$

$$\frac{35}{\sin \angle CAD} \doteq \frac{135.922\ 772\ 22}{\sin 60°}$$

$$\sin \angle CAD \doteq \frac{35 \sin 60°}{135.922\ 772\ 22}$$

$$\angle CAD \doteq \sin^{-1}\left(\frac{35 \sin 60°}{135.922\ 772\ 22}\right)$$

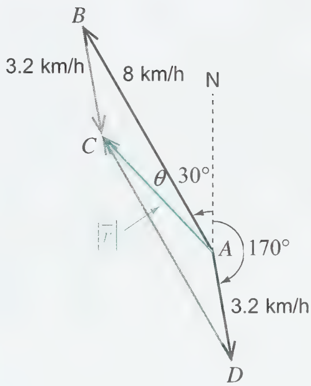
$$\doteq 13°$$



$$\therefore \text{ Bearing} \doteq 225° + 13°$$
$$\doteq 238°$$

The aircraft's resultant velocity is approximately 135.9 km/h [238°].

8. $\angle BAD = 330° - 170°$
$= 160°$



$\therefore \angle ABC = 180° - 160°$
$= 20°$

Use the Cosine Law to determine the resultant velocity of the kayak.

$$|\vec{r}|^2 = a^2 + c^2 - 2ac \cos \angle ABC$$

$$|\vec{r}|^2 = (3.2)^2 + (8)^2 - 2(3.2)(8)\cos 20°$$

$$|\vec{r}|^2 \doteq 26.127\ 737\ 82$$

$$|\vec{r}| \doteq 5.111\ 529\ 89$$
$$\doteq 5.1 \text{ km/h}$$

Use the Sine Law to determine $\angle BAC$.

$$\frac{BC}{\sin \angle BAC} = \frac{AC}{\sin \angle ABC}$$

$$\frac{3.2}{\sin \angle BAC} \doteq \frac{5.111\,529\,89}{\sin 20^\circ}$$

$$\sin \angle BAC \doteq \frac{3.2 \sin 20^\circ}{5.111\,529\,89}$$

$$\angle BAC \doteq \sin^{-1}\left(\frac{3.2 \sin 20^\circ}{5.111\,529\,89}\right)$$

$$\doteq 12^\circ$$

$$\therefore \text{ Bearing} \doteq 330^\circ - 12^\circ$$

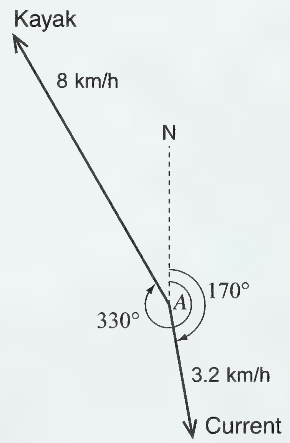
$$\doteq 318^\circ$$

The resultant velocity of the kayak is about 5.1 km/h [318°].

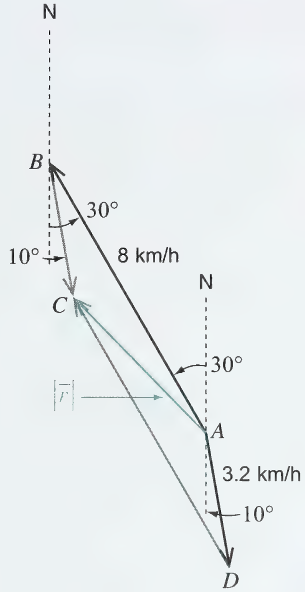
8. Textbook exercise "Communicating the Ideas," p. 329

Answers will vary. A detailed description of how to solve exercise 8 on page 329 of the textbook is given.

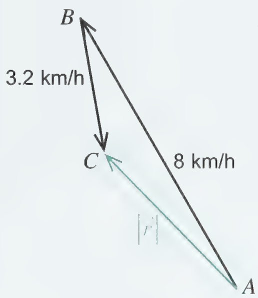
Step 1: Sketch the situation and all the vectors involved.



Step 2: Sketch the corresponding vector diagram.



Step 3: Extract a triangle from the vector sketch, and determine the angle between the two known sides. Show the angle and side you need to determine.



$$\begin{aligned}\angle ABC &= 30° - 10° \\ &= 20°\end{aligned}$$

Refer to the sketch in Step 2.

Step 4: Use the Cosine Law to determine the unknown side.

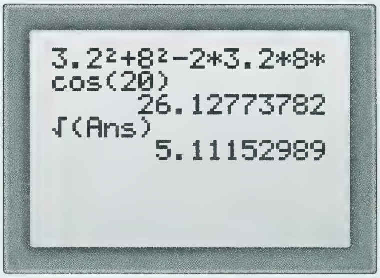
$$|\vec{r}|^2 = a^2 + c^2 - 2ac\cos B$$

$$|\vec{r}|^2 = (3.2)^2 + (8)^2 - 2(3.2)(8)\cos 20^\circ$$

$$|\vec{r}|^2 \doteq 26.127\,737\,82$$

$$|\vec{r}| \doteq 5.111\,529\,89$$

$$\doteq 5.1 \text{ km/h}$$



Step 5: Use the Sine Law to determine the unknown angle. **Remember:** Use the unrounded value of side B.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

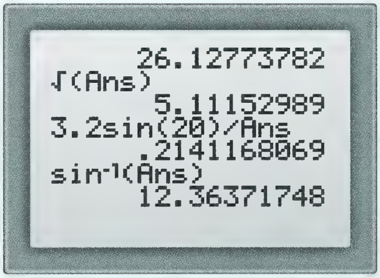
$$\frac{3.2}{\sin A} \doteq \frac{5.111\,529\,89}{\sin 20^\circ}$$

$$\sin A \doteq \frac{3.2\sin 20^\circ}{5.111\,529\,89}$$

$$\sin A \doteq 0.214\,116\,806\,9$$

$$\angle A \doteq \sin^{-1}(0.214\,116\,806\,9)$$

$$\doteq 12^\circ$$



Step 6: Determine the bearing of the resultant.

$$\text{Bearing} \doteq 330^\circ - 12^\circ$$

$$\doteq 318^\circ$$

Step 7: Write a concluding statement.

The resultant velocity of the kayak is approximately 5.1 km/h [318°].

Activity 5: Vector Problems in Three Dimensions

1. You would have to draw a three-dimensional shape on the paper so that it appears that the vectors are in three dimensions.

2. **Textbook exercises 1 to 5 of "Investigation 1: Displacements in Three Dimensions," pp. 330 and 331**

Answers may vary. Sample answers are given.

1. length $= 34$ cm, width $= 20$ cm, and height $= 14$ cm

2. **a. to c.** Your drawing should be similar to the one in the textbook on page 330.

3. Since $\vec{d}_{12}$, $\vec{d}_1$, and $\vec{d}_2$ are sides of a right triangle, you can determine the length of $\vec{d}_{12}$ using the Pythagorean Theorem.

$$\left(\vec{d}_{12}\right)^2 = \left(\vec{d}_1\right)^2 + \left(\vec{d}_2\right)^2$$
$$\left(\vec{d}_{12}\right)^2 = (20)^2 + (34)^2$$
$$\left(\vec{d}_{12}\right)^2 = 400 + 1156$$
$$\left(\vec{d}_{12}\right)^2 = 1556$$
$$\vec{d}_{12} = \sqrt{1556}$$
$$\doteq 39.446\,165\,85$$

$$\tan(\angle BAC) = \frac{AB}{BC}$$
$$\tan(\angle BAC) = \frac{34}{20}$$
$$\angle BAC = \tan^{-1}\left(\frac{34}{20}\right)$$
$$\doteq 60°$$

$$\therefore \text{Bearing} \doteq 90° - 60°$$
$$\doteq 30°$$

$$\therefore\ \vec{d}_{12} \doteq 39.4 \text{ cm } [030°]$$

4. **a.** The measure of $\angle ACG$ is 90°.

b. Vectors $\vec{d}_{12}$ and $\vec{d}_3$ are two of the edges of rectangle $ACGE$.

Activity 5 (continued)

c. Use the Pythagorean Theorem.

$$\left(\vec{d}\right)^2 = \left(\vec{d}_{12}\right)^2 + \left(\vec{d}_3\right)^2$$
$$\left(\vec{d}\right)^2 \doteq 1556 + (14)^2$$
$$\left(\vec{d}\right)^2 \doteq 1556 + 196$$
$$\left(\vec{d}\right)^2 \doteq 1752$$
$$\vec{d} \doteq 41.856\,899\,07$$
$$\doteq 41.9 \text{ cm}$$

Notice that the exact value of $\left(\vec{d}_{12}\right)^2$ was used.

Determine the direction.

$$\tan(\angle CAG) = \frac{CG}{AC}$$
$$\tan(\angle CAG) \doteq \frac{14}{39.446\,165\,85}$$
$$\angle CAG \doteq \tan^{-1}\left(\frac{14}{39.446\,165\,85}\right)$$
$$\doteq 19.540\,490\,76$$
$$\doteq 20°$$

The direction of $\vec{d}$ is the same as the direction of $\vec{d}_{12}$, except $\vec{d}$ has a vertical direction as well.

Therefore, $\vec{d}$ is about 41.9 cm bearing 30° east of north and 20° up.

5. Vector $\vec{d}_{12}$, is the resultant of $\vec{d}_1$ and $\vec{d}_2$. The direction of $\vec{d}_{12}$ was determined to be about 30° east of north.

Vector $\vec{d}$ is the resultant of $\vec{d}_{12}$, and $\vec{d}_3$. The direction of $\vec{d}$ from north is the same as the direction of $\vec{d}_{12}$ from north (30°). The direction of $\vec{d}$, however, also had a vertical direction. It was determined to be about 30° east of north and 20° up.

3. Textbook exercises 1 and 2 of "Exercises: Checking Your Skills," p. 336

1. Two of the forces are parallel to each other: the force of gravity (30 000 N [vertically down]) and the lift force created by the rotors (33 000 N [vertically up]). These forces can be combined directly to find the net vertical force on the helicopter.

$$30\ 000\ \text{N}\left[\text{down}\right]+33\ 000\ \text{N}\left[\text{up}\right]=3000\ \text{N}\left[\text{up}\right]$$

The net vertical force is 3000 N [vertically up].

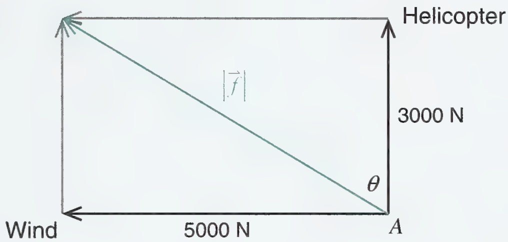
Add the net vertical force and the wind force to find the resultant force.

$$\therefore\ |\vec{f}|^2 = 3000^2 + 5000^2$$

$$|\vec{f}|^2 = 9\ 000\ 000 + 25\ 000\ 000$$

$$|\vec{f}|^2 = 34\ 000\ 000$$

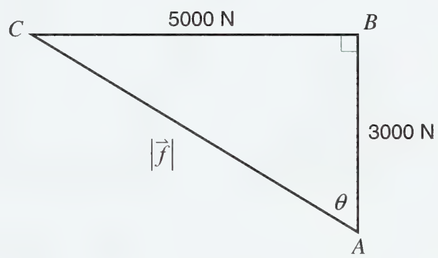
$$|\vec{f}| = \sqrt{34\ 000\ 000}$$
$$\doteq 5831.0\ \text{N}$$



$$\tan A = \frac{BC}{AB}$$

$$\tan A = \frac{5000}{3000}$$

$$\angle A = \tan^{-1}\left(\frac{5000}{3000}\right)$$
$$\doteq 59.0°$$



The net force on the helicopter is about 5831.0 N, 59.0° away from the vertical.

2. Determine the net resultant force of the two dogs pulling.

$$100\ \text{N}\left[\text{toward Buster}\right]+80\ \text{N}\left[\text{toward Koko}\right]=20\ \text{N}\left[\text{toward Buster}\right]$$

The net sideways force is 20 N [toward Buster].

Activity 5 (continued)

$$\therefore\ |\vec{f}|^2 = (BC)^2 + (AB)^2$$

$$|\vec{f}|^2 = (20)^2 + (175)^2$$

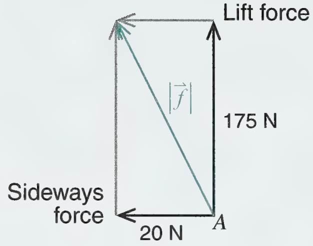
$$|\vec{f}|^2 = 400 + 30\,625$$

$$|\vec{f}|^2 = 31\,025$$

$$|\vec{f}| = \sqrt{31\,025}$$

$$\doteq 176.139\,149\,5$$

$$\doteq 176.1\text{ N}$$



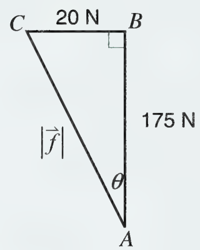
$$\tan A = \frac{BC}{AB}$$

$$\tan A = \frac{20}{175}$$

$$\angle A = \tan^{-1}\left(\frac{20}{175}\right)$$

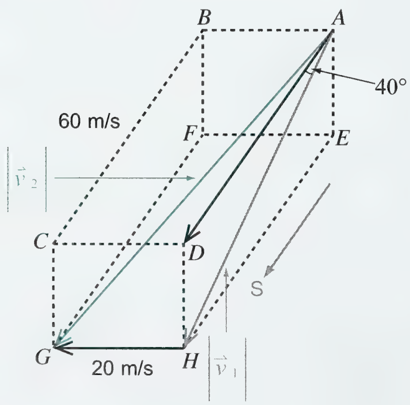
$$\doteq 6.519\,801\,752$$

$$\doteq 6.5°$$

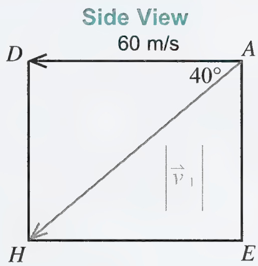


The net force on the stick is about 176.1 N, 6.5° from vertical toward Buster.

4. a.



b. To determine the final resultant, $\left|\vec{v}_2\right|$, you must first determine $\left|\vec{v}_1\right|$.

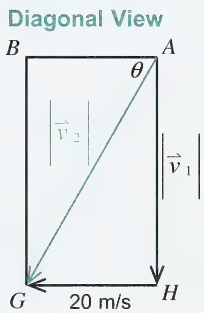


$$\cos \angle DAH = \frac{AD}{AH}$$

$$\cos 40° = \frac{60}{\left|\vec{v}_1\right|}$$

$$\left|\vec{v}_1\right| = \frac{60}{\cos 40°}$$

$$\doteq 78.324\ 437\ 36$$



$$\therefore \left|\vec{v}_2\right|^2 = \left|\vec{v}_1\right|^2 + (GH)^2$$

$$\left|\vec{v}_2\right|^2 \doteq (78.324\ 437\ 36)^2 + (20)^2$$

$$\left|\vec{v}_2\right|^2 \doteq 6534.717\ 488$$

$$\left|\vec{v}_2\right| \doteq 80.837\ 599\ 47$$

The magnitude of the resultant velocity is about 80.8 m/s.

c. The angle the resultant makes with the horizontal, $\angle CAG$, is determined using ΔAGC.

First, determine *CG*.

$$\tan \angle DAH = \frac{DH}{AD}$$

$$\tan 40° = \frac{DH}{60}$$

$$DH = 60 \tan 40°$$

$$CG = DH$$

$$= 60 \tan 40°$$

Now determine $\angle CAG$.

$$\sin \angle CAG = \frac{CG}{\left|\vec{v}_2\right|}$$

$$\sin \angle CAG \doteq \frac{60 \tan 40°}{80.837\ 599\ 47}$$

$$\angle CAG \doteq \sin^{-1}\left(\frac{60 \tan 40°}{80.837\ 599\ 47}\right)$$

$$\doteq 38.521\ 185\ 59°$$

The angle at which the helicopter is dropping is about 39°.

d. The bearing of the helicopter is determined by the direction of AC (the projection of $\overrightarrow{AG}$ on the horizontal).

$$\tan \angle CAD = \frac{CD}{AD}$$

$$\tan \angle CAD = \frac{20}{60}$$

$$\angle CAD = \tan^{-1}\left(\frac{20}{60}\right)$$

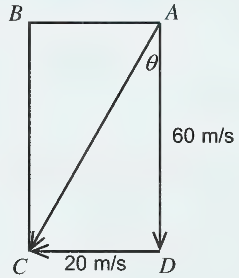
$$\doteq 18.434\ 948\ 82°$$

$$\doteq 18°$$

$$\therefore \text{Bearing} \doteq 180° + 18°$$

$$\doteq 198°$$

Top View



e. The resultant velocity of the helicopter is approximately 80.8 m/s, dropping at an angle of 39° at a bearing of 198°.

5. Textbook exercise "Communicating the Ideas," p. 337

To solve a three-dimensional vector problem, begin by reading the problem carefully and make note of the given information. Next, draw a three-dimensional box and write the given information in the appropriate locations. Determine the magnitude of the resultant using the Pythagorean Theorem. If the problem is one involving flight in space, determine the angle the resultant makes with the horizontal using the inverse tangent ratio with the appropriate measures. Also, determine the bearing using the tangent ratio with the appropriate measures.

Module Review

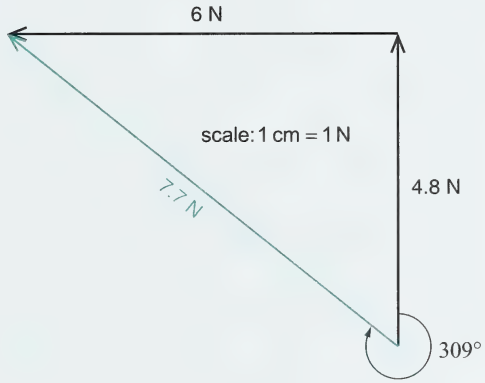
Textbook exercises 2.b., 2.d., 4, 6, and 7 of Part B of "What Should I Be Able to Do?," pp. 341 and 342

2. b.

$\vec{c}-\vec{d}=6\ \text{N}\,[\text{west}]$

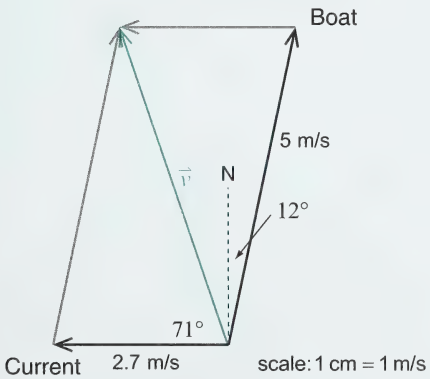
d. $1.2\vec{b}=1.2\times 4\ \text{N}\,[\text{north}]$ $\quad 3\vec{c}=3\times 2\ \text{N}\,[\text{west}]$

$=4.8\ \text{N}\,[\text{north}]$ $\quad =6\ \text{N}\,[\text{west}]$



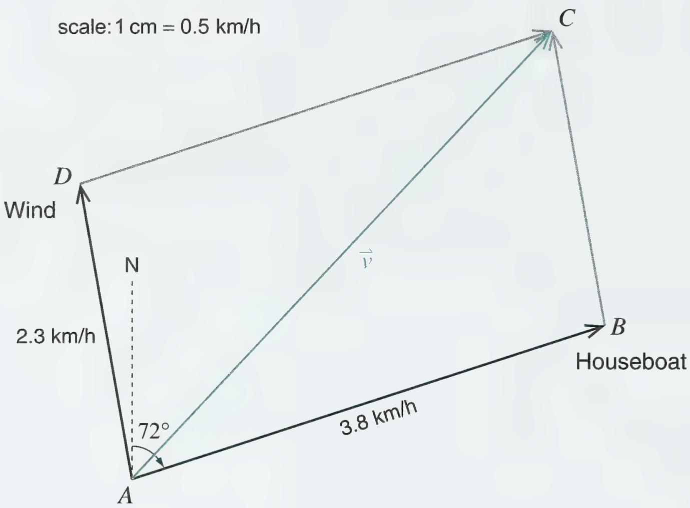
$\therefore\ 1.2\vec{b}+3\vec{c}=7.7\ \text{N}\,[309°]$

4.



The resultant velocity is 5.2 m/s [341°].

6. **Methold 1: Using a Scale Diagram**



$9.4\ \text{cm} \times 0.5\ \text{km/h/cm} = 4.7\ \text{km/h}$

The resultant velocity is 4.7 km/h [043°].

Method 2: Using Trigonometry

$$
\begin{aligned}
\angle ABC &= 180° - \angle BAD \\
&= 180° - (72° + 10°) \\
&= 180° - 82° \\
&= 98°
\end{aligned}
$$

Use the Cosine Law to determine $\vec{v}$.

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$\vec{v}^{\,2} = (2.3)^2 + (3.8)^2 - 2(2.3)(3.8)\cos 98°$$

$$\vec{v}^{\,2} \doteq 22.162\ 745\ 8$$

$$
\begin{aligned}
\vec{v} &\doteq 4.707\ 732\ 555 \\
&\doteq 4.7\ \text{km/h}
\end{aligned}
$$

Determine the bearing.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{2.3}{\sin A} \doteq \frac{4.707\ 732\ 555}{\sin 98°}$$

$$\sin A \doteq \frac{2.3 \sin 98°}{4.707\ 732\ 555}$$

$$A \doteq \sin^{-1}\left(\frac{2.3 \sin 98°}{4.707\ 732\ 555}\right)$$

$$\doteq 28.934\ 097\ 77$$

$$\doteq 29°$$

$$\therefore \text{ Bearing} \doteq 72° - 29°$$

$$\doteq 43°$$

The resultant velocity is approximately 4.7 km/h [043°].

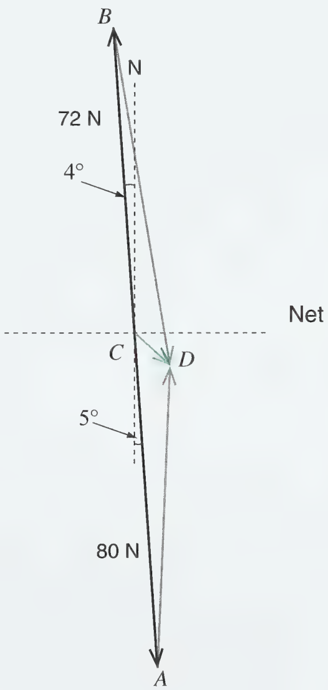
7. a. Because the vectors are almost opposite each other, draw the diagram so the angles are exaggerated.

$$\angle CAD = 180° - \angle ABC$$

$$= 180° - (4° + 175°)$$

$$= 180° - 179°$$

$$= 1°$$



Use the Cosine Law to determine $\vec{f}$.

$$a^2 = c^2 + d^2 - 2cd \cos A$$

$$\vec{f}^{\,2} = (72)^2 + (80)^2 - 2(72)(80)\cos 1°$$

$$\vec{f}^{\,2} \doteq 65.754\ 551\ 8$$

$$\vec{f} \doteq \sqrt{65.754\ 551\ 8}$$

$$\doteq 8.108\ 918\ 041$$

$$\doteq 8.1 \text{ N}$$

Use the Sine Law to determine $\angle ACD$.

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{8.108\,918\,041}{\sin 1°} \doteq \frac{72}{\sin C}$$

$$\sin C \doteq \frac{72 \sin 1°}{8.108\,918\,041}$$

$$C \doteq \sin^{-1}\left(\frac{72 \sin 1°}{8.108\,918\,041}\right)$$

$$\doteq 8.914\,585\,991$$

$$\doteq 9°$$

$\therefore$ Bearing $\doteq 175° - 9°$

$\doteq 166°$

The resultant force on the ball is about 8.1 N [166°].

b. Since the bearing is between [090°] and [180°], the ball falls on player B's side of the court.

Enrichment

1. $\vec{a} = [-3, 8]$

$\vec{b} = [12, 8]$

$\vec{c} = [7, 1]$

$\vec{d} = [4, -4]$

$\vec{e} = [-2, -5]$

2. **a.** $\vec{a} + \vec{d} = [-3, 8] + [4, -4]$

$= [-3 + 4, 8 + (-4)]$

$= [1, 4]$

b. $\vec{c}+\vec{e}=[7,1]+[-2,-5]$
$=[7+(-2),1-5]$
$=[5,-4]$

c. $\vec{b}-\vec{d}=[12,8]-[4,-4]$
$=[12-4,8-(-4)]$
$=[8,12]$

3. x-coordinate $=4-(-6)$
$=10$

y-coordinate $=6-8$
$=-2$

$\therefore\ \vec{s}=[10,-2]$

x-coordinate $=9-1$
$=8$

y-coordinate $=-3-4$
$=-7$

$\therefore\ \vec{h}=[8,-7]$

x-coordinate $=1-(-8)$
$=9$

y-coordinate $=2-4$
$=-2$

$\therefore\ \vec{m}=[9,-2]$

4. **a.** Magnitude $=\sqrt{x^2+y^2}$
$=\sqrt{3^2+4^2}$
$=\sqrt{9+16}$
$=\sqrt{25}$
$=5$

Direction $=\tan^{-1}\left(\frac{x}{y}\right)$
$=\tan^{-1}\left(\frac{3}{4}\right)$
$\doteq 36.869\,897\,65°$
$\doteq 37°$

Vector $[3,4]$ has a magnitude of 5 and a direction of about 37°.

b. Magnitude $=\sqrt{x^2+y^2}$
$=\sqrt{(4)^2+(-1)^2}$
$=\sqrt{16+1}$
$=\sqrt{17}$
$\doteq 4.123\,105\,626$
$\doteq 4.1$

Direction $=\tan^{-1}\left(\frac{x}{y}\right)$
$=\tan^{-1}\left(\frac{4}{-1}\right)$
$\doteq -75.963\,756\,53$
$\doteq -76°$ or $284°$

Vector $[4,-1]$ has a magnitude of about 4.1 and a direction of about 284°.

5. a. $[r \times \sin\theta, r \times \cos\theta] = [8 \sin 60°, 8 \cos 60°]$

$$\doteq [6.928\,203\,23, 4]$$

$$\doteq [6.9, 4]$$

b. $[r \times \sin\theta, r \times \cos\theta] = [3.25 \sin 325°, 3.25 \cos 325°]$

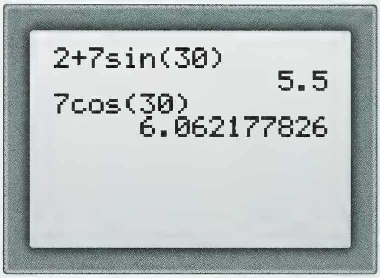
$$\doteq [-1.864\,123\,418, 2.662\,244\,144]$$

$$\doteq [-1.86, 2.66]$$

6. Change magnitude 7 [030°] to bracket form.

$[r \times \sin\theta, r \times \cos\theta] = [7 \sin 30°, 7 \cos 30°]$ or about $[3.50, 6.06]$

Add the four bracket form vectors.



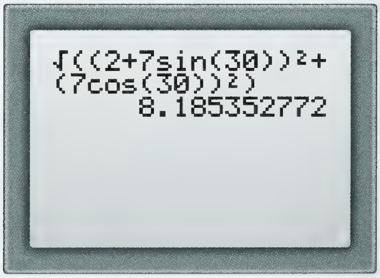
$$[1, 3] + [-7, 4] + [8, -7] + [7 \sin 30°, 7 \cos 30°]$$

$$= [1 + (-7) + 8 + 7 \sin 30°, 3 + 4 + (-7) + 7 \cos 30°]$$

$$= [2 + 7 \sin 30°, 7 \cos 30°]$$

$$\doteq [5.5, 6.06]$$

The sum of the four vectors is about $[5.50, 6.06]$.



$$\text{Magnitude} = \sqrt{x^2 + y^2}$$

$$= \sqrt{(2 + 7 \sin 30°)^2 + (7 \cos 30°)^2}$$

$$= \sqrt{4 + 28 \sin 30° + 49 \sin^2 30° + 49 \cos^2 30°}$$

$$= \sqrt{4 + 28 \sin 30° + 49(\sin^2 30° + \cos^2 30°)}$$

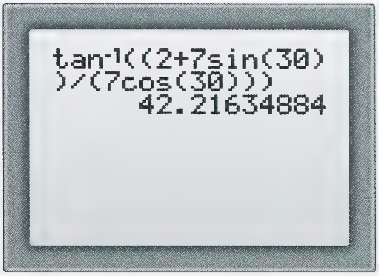
$$= \sqrt{4 + 14 + 49}$$

$$= \sqrt{67}$$

$$\doteq 8.185\,352\,772$$

Note: $(\sin^2 30° + \cos^2 30°) = 1$

$$\text{Direction} = \tan^{-1}\left(\frac{x}{y}\right)$$
$$= \tan^{-1}\left(\frac{2+7\sin 30°}{7\cos 30°}\right)$$
$$\doteq 42.22°$$



The sum of the four vectors has a magnitude of about 8.19 and a bearing of 42.22°.

Module Project: Accident Reconstruction

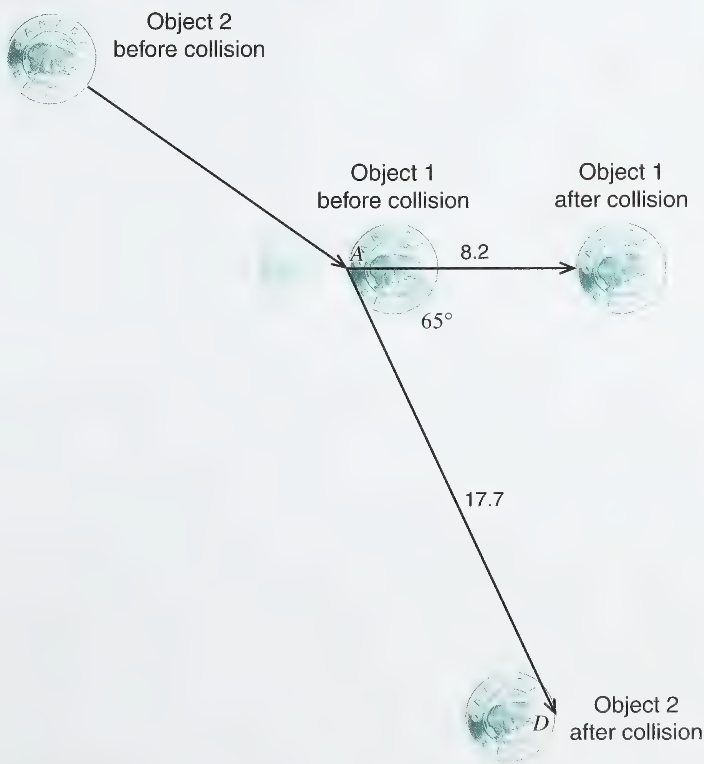
Completing the Project

1. Textbook exercises 1 to 6 of "Simulating an Accident," pp. 310 and 311

Answers will vary. A sample simulation is given.

1. to 6. In this simulation of an accident, the two objects used were Canadian $2 coins (toonies). Two coins were taped together for Object 1 so it was double the mass of Object 2, which consists of only one toonie.

The following diagram was obtained from the simulation.



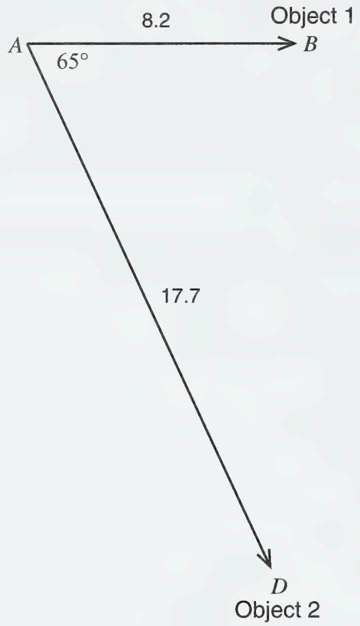
Module Project (continued)

2. Textbook exercise 10 of Part C of "What Should I Be Able to Do?," pp. 343 and 344

10. The student's calculations are correct and generally follow a logical order. There is no explanation of how momentum is used, but the calculations show that the student understands what is required. The numbering does not match the textbook numbering. The student appears to have answered exercises 8 and 9 on pages 339 and numbered these exercises as 4 and 5. The accident report is weak and does not explain how vectors and momentum are used in accident reconstruction.

3. Textbook exercises 1 to 4 of "How Fast Was That Car Going?," p. 338

1. scale: 1 cm = 2 cm



2. $\vec{v}_1 = \sqrt{8.2}$ $\doteq 2.9$

$\vec{v}_2 = \sqrt{17.7}$ $\doteq 4.2$

The velocity of Object 1 before the collision was 0, and the velocity of Object 2 before the collision was $\vec{v}$.

Momentum before collision = Momentum after collision

$$\text{Object 1} + \text{Object 2} = \text{Object 1} + \text{Object 2}$$

$$0 + m_2\vec{v} = m_1\vec{v}_1 + m_2\vec{v}_2$$

$$m_2\vec{v} = (2m_2)\vec{v}_1 + m_2\vec{v}_2 \quad \longleftarrow m_1 = 2m_2$$

$$\vec{v} = \frac{2m_2\vec{v}_1 + m_2\vec{v}_2}{m_2}$$

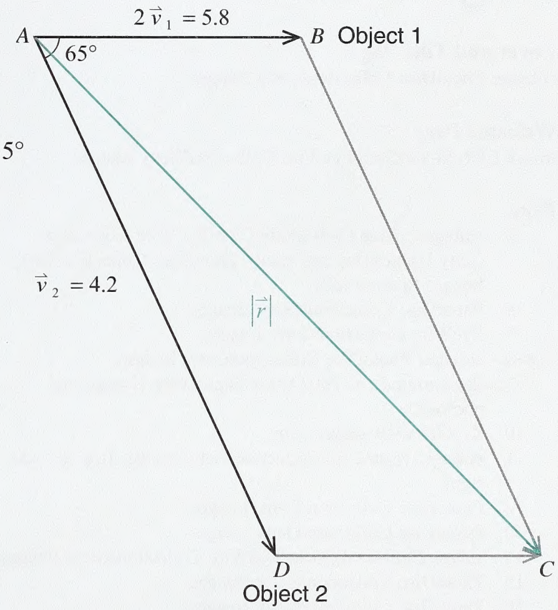
$$= \frac{\cancel{m_2}\left(2\vec{v}_1 + \vec{v}_2\right)}{\cancel{m_2}}$$

$$= 2\vec{v}_1 + \vec{v}_2$$

3. $2\vec{v}_1 \doteq 2(2.9)$

$\doteq 5.8$

$$\angle ABC = 180° - \angle BAD$$
$$= 180° - 65°$$
$$= 115°$$



$$\therefore\ |\vec{r}|^2 = (5.8)^2 + (4.2)^2 - 2(5.8)(4.2)\cos 115°$$

$$|\vec{r}|^2 \doteq 71.869\,961\,71$$

$$|\vec{r}| \doteq 8.477\,615\,332$$
$$\doteq 8.5$$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{4.2}{\sin \angle BAC} \doteq \frac{8.477\,615\,332}{\sin 115°}$$

$$\sin \angle BAC \doteq \frac{4.2 \sin 115°}{8.477\,615\,332}$$

$$\angle BAC \doteq \sin^{-1}\left(\frac{4.2 \sin 115°}{8.477\,615\,332}\right)$$
$$\doteq 26.7°$$

The resultant of $2\vec{v}_1 + \vec{v}_2$ is about 8.5 bearing 26.7° below the path of Object 1 after the collision.

Module Project (continued)

4. Since $\vec{v} = 2\vec{v}_1 + \vec{v}_2$, the velocity of Object 2 before it collided with Object 1 was about 8.5. Therefore, the momentum before the collision was $8.5m_2$, where m_2 is the mass of Object 2.

After the collision, the momentum of each object was as follows:

$$\begin{aligned}\text{Momentum of Object } 1 &= m_1 \times 2.9 \\ &= 2.9m_1 \\ &= 2.9(2m_2) \quad \longleftarrow m_1 = 2m_2 \\ &= 5.8m_2\end{aligned}$$

$$\begin{aligned}\text{Momentum of Object } 2 &= m_2 \times 4.2 \\ &= 4.2m_2\end{aligned}$$

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